Precalculus

Lesson 14.1: Finding Limits Using Tables and Graphs

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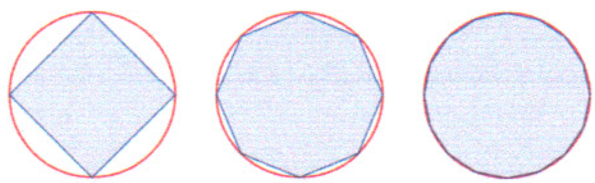
The title of this chapter is "Limits: A Preview of Calculus." The central idea behind calculus is the concept of a **limit.** Calculus is used in modeling numerous real-life phenomena, particularly situations that involve change or motion. To better understand limits let's look back at the Greeks some 2500 years ago and how they used the "method of exhaustion" to find areas. To find the area of a circle for example, the Greeks inscribed a polygon inside the curved region. As the number of sides of the polygon increases, the polygon's area approaches the area of the circle. In other words:

$$area = \lim_{n\to\infty} a_n$$



n=4 n=8 n=16

Definition of a Limit:

We write:

$$\lim_{x\to a} f(x) = L$$

We say: The limit of f(x), as x approaches a, equals L.

We mean: as x gets closer and closer to a, the y value gets closer and closer to L.

Finding a Limit From a Table

Find the limit of

$$\lim_{x\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$$

FROM THE RIGHT

t	0.5	0.1	0.01	0.001
f(x)	.16553	.16662	.16667	.16667

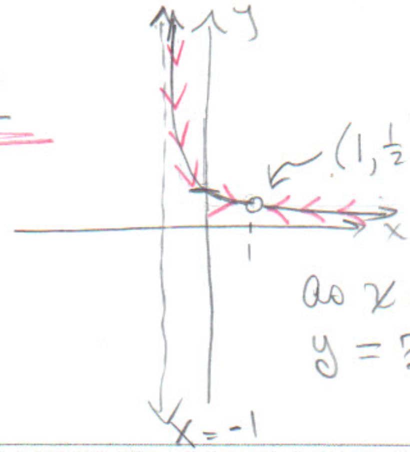
FROM THE LEFT

t	−0.5	−0.1	−0.01	−0.001
f(x)	.16653	.16662	.16667	.16667

Find the limit from a table and estimate using the graph of (what are the restrictions?):

$$\lim_{x\to1}\frac{x-1}{x^2-1} = \frac{1}{2}$$

x	f(x)
1.1	.48
1.01	.497
1.001	.5
.9	.526
.99	.50



$$\frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{1}{x+1}$$

hole at x = 1

∴ $(1, \frac{1}{2})$

as x approaches 1

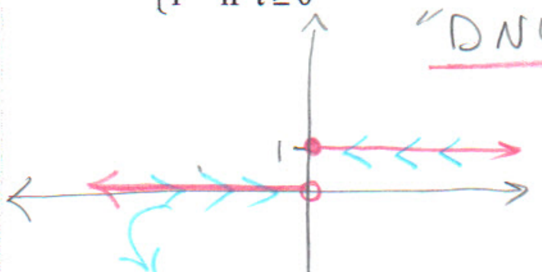
y = ?

Limits That Fail to Exist: A Function With a Jump

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

limit does not exist => limit from left is different from limit from right.

"DNE"



as "t" approaches 0 from right y = 1

as "t" approaches 0 from left y = 0

Limits That Fail to Exist: A Function That Oscillates

$$\lim_{x\to0}\sin\frac{\pi}{x} \Rightarrow DNE$$

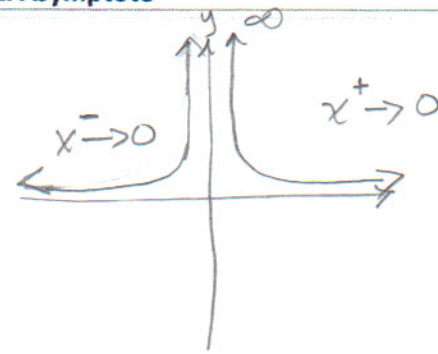
x	f(x)
$\frac{1}{2}$	$\sin\frac{\pi}{\frac{1}{2}} = \sin 2\pi = 0$
$\frac{1}{6}$	$\sin 6\pi = 0$
$-\frac{1}{6}$	$\sin -6\pi = 0$
$\pm\frac{1}{10}$	$\sin \pm 10\pi = 0$

x	f(x)
.06	.866
.006	.866
-.06	-.866
-.006	-.866

Oscillates = no set value

Limits That Fail to Exist: A Function with a Vertical Asymptote

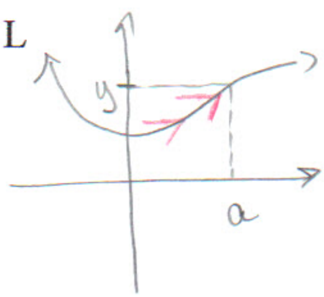
$\lim_{x\to 0}\frac{1}{x^2} = \infty \therefore \text{DNE}$



One Sided Limits

Left Sided Limit

$\lim_{x\to a^-} f(x) = L$

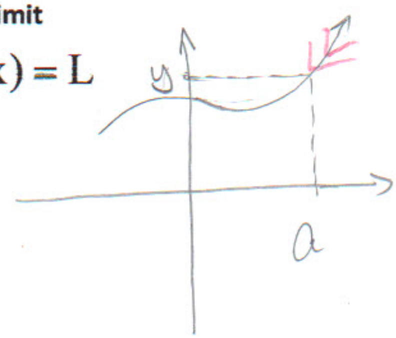


$x^- \to a$

x approaches "a" from negative side

Right Sided Limit

$\lim_{x\to a^+} f(x) = L$



$x^+ \to a$

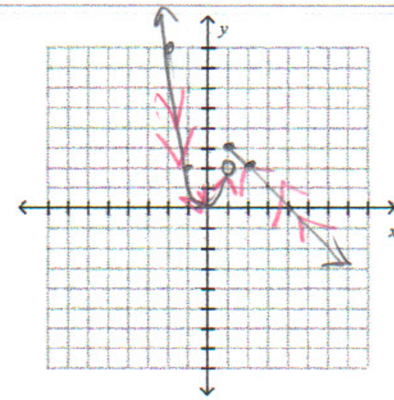
x approaches "a" from positive side

Very Important!!!!

$\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$ AND $\lim_{x\to a^+} f(x) = L$

Limits From a Graph: A Piecewise-Defined Function

$$f(x)=\begin{cases}2x^2 & \text{if } x<1\\ 4-x & \text{if } x\geq 1\end{cases}$$



note ☆ When $x < 1$ we are on $f(x) = 2x^2$ branch

When $x \geq 1$ we are on $f(x) = 4-x$ branch

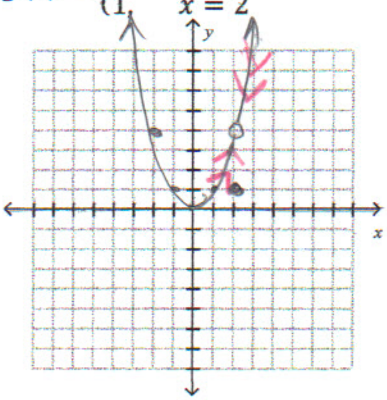
Find:

$\lim_{x\to 1^-} f(x)$ $= 2$

$\lim_{x\to 1^+} f(x)$ $= 3$

$\lim_{x\to 1} f(x)$ DNE (jump function)

$$g(x)=\begin{cases}x^2, & x\neq 2\\ 1, & x=2\end{cases}$$



find $\lim_{x\to 2} g(x)$ $= 4$

$\lim_{x\to 2^-} g(x) = 4$ $\lim_{x\to 2^+} g(x) = 4$

as we approach from + & − we are on $f(x) = x^2$ curve