Precalculus

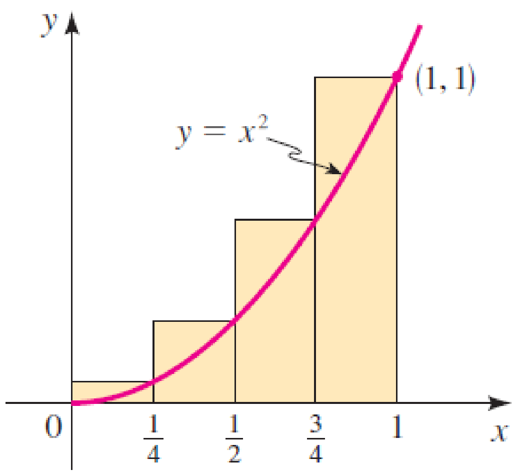
Lesson 14.5: The Area Problem: The integral

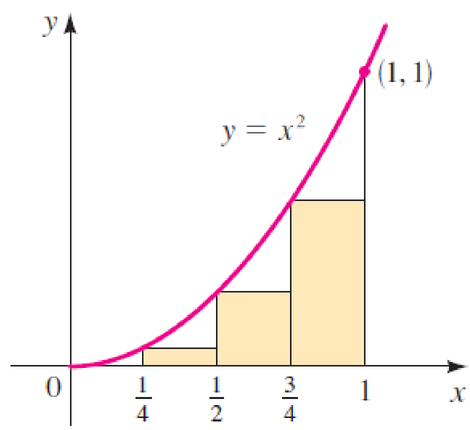
Mrs. Snow, Instructor

In geometry we found area of polygons. We had set formulas such as the area of a rectangle is length times width. A triangular area is found by calculating ½ the length of the base times the height, and so on. Calculus is used to deal with area problems that have regions containing curved boundaries. Here we can go back to our simple formula for the area of a rectangle and use it to estimate the area of a region under a curve.

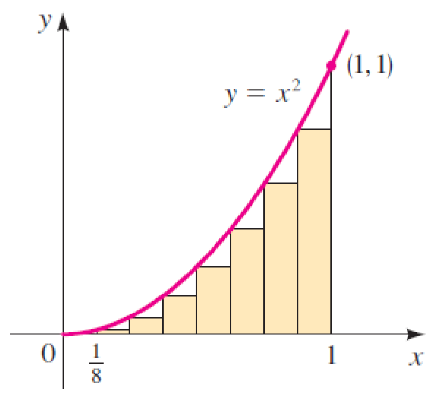
Estimating an Area Using Rectangles

Use rectangles to estimate the area under the parabola from 0 to 1.



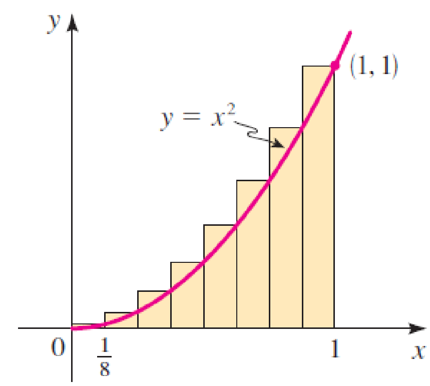


Same problem….smaller rectangles



(a) Using left endpoints

Underestimate of area.



(b) Using right endpoints

Overestimate of area.

The smaller the rectangular strips the more accurate the calculation of the area. This then opens up the door to take a limit as the number of rectangles goes to infinity.

Definition of Area

The area A of the region S that lies under the graph of a continuous function f is the limit of the sum of the areas of the approximating rectangles: ***use right endpoints***.

AREA=(height) x (width)

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

Δx **is the width** of an approximating rectangle,
x_k **is the right endpoint** of the kth **rectangle**
$f(x_k)$ **is its height**.

$\boldsymbol{n}$ $rectangles$
$region\ from\ x = a \quad to \quad x = b$

$$\boldsymbol{width}: \qquad \Delta \boldsymbol{x} = \frac{\boldsymbol{b-a}}{\boldsymbol{n}}$$

$$\boldsymbol{right\ endpoint}: \qquad \boldsymbol{x_k = a + k\Delta x}$$

$$\boldsymbol{height}: \qquad \boldsymbol{f(x_k) = f(a + k\Delta x)}$$

The summation formulas we studied are now applied to finding the area under a curve:

Summation Formulas!!

these will be used in solving of our area problems

$$\sum_{k=1}^{n} c = nc$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Finding the Area under a Curve:

Find the area of the region that lies under

$y = x^2,\ 0 \le x \le 5$

Finding an Area under a Curve

Find the area of the region that lies under

$y = 4x - x^2 \quad 1 \leq x \leq 3$