Chapter 12 Review Solutions

1) 56

2) $s_1 = 4$, $s_2 = 10$, $s_3 = 16$, $s_4 = 22$, $s_5 = 28$

3) $c_1 = 4$, $c_2 = 8$, $c_3 = \frac{64}{3}$, $c_4 = 64$, $c_5 = \frac{1024}{5}$

4) $a_n = 2n - 3$

5) $a_n = (-1)^{n+1} \cdot 4n$

6) $a_1 = 5$, $a_2 = 3$, $a_3 = 1$, $a_4 = -1$

7) $3 + 4 + 5 + ... + (n + 2)$

8) $\sum_{k=3}^{10} k^2$

9) 15

10) $d = -5$; $s_1 = 4$, $s_2 = -1$, $s_3 = -6$, $s_4 = -11$

11) $a_n = 94 - 10n$; $a_8 = 14$

12) 220

13) $a_1 = 73$, $d = -5$, $a_n = a_{n-1} - 5$

14) 34980

15) 4774

16) $r = 3$; $s_1 = 3$, $s_2 = 9$, $s_3 = 27$, $s_4 = 81$

17) Geometric; $r = -3$

18) $a_5 = 2500$; $a_n = 4 \cdot (5)^{n-1}$

19) $a_8 = 128$

20) $a_n = 7 \cdot 2^{n-1}$

21) $a_n = 3(3)^{n-1}$

22) 726

23) Converges; $\frac{9}{2}$

24) First we show that the statement is true when n = 1.

For n = 1, we get $2 = \frac{(1)}{2}(3(1) + 1) = 2$.

This is a true statement and Condition I is satisfied.

Next, we assume the statement holds for some k. That is, $2 + 5 + 8 + ... + (3k - 1) = \frac{k}{2}(3k + 1)$ is true for some positive integer k.

We need to show that the statement holds for k + 1. That is, we need to show that

$$2 + 5 + 8 + ... + (3(k + 1) - 1) = \frac{k+1}{2}(3(k + 1) + 1).$$

So we assume that $2 + 5 + 8 + ... + (3k - 1) = \frac{k}{2}(3k + 1)$ is true and add the next term, $3(k + 1) - 1$, to both sides of the equation.

$$2 + 5 + 8 + ... + (3k - 1) + 3(k + 1) - 1 = \frac{k}{2}(3k + 1) + 3(k + 1) - 1$$

$$= \frac{1}{2}[k(3k + 1) + 6(k + 1) - 2]$$

$$= \frac{1}{2}(3k^2 + k + 6k + 6 - 2)$$

$$= \frac{1}{2}(3k^2 + 7k + 4)$$

$$= \frac{1}{2}(k + 1)(3k + 4)$$

$$= \frac{k+1}{2}(3k + 3 + 1)$$

$$= \frac{k+1}{2}(3(k + 1) + 1)$$

Condition II is satisfied. As a result, the statement is true for all natural numbers n.

25) 10

26) $625x^4 - 1000x^3 + 600x^2 - 160x + 16$

27) 972x