Precalculus
Lesson 12.4: Mathematical Induction
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Mathematical induction is a method for proving that statements involving natural numbers are true for all natural numbers.

The Principle of Mathematical Induction

Suppose that the following two conditions are satisfied with regard to a statement about natural numbers:

CONDITION I: The statement is true for the natural number 1.

CONDITION II: If the statement is true for some natural number k, it is also true for the next natural number $k+1$.

Then the statement is true for all natural numbers.

translation:
#1 show statement is true for ***n=1***
#2 assume statement is true for ***n=k,***
now show statement is true for ***n=k+1*** *∴ true for all numbers*

Show that the following statement is true for all natural numbers n.

$$1+3+5+\cdots+(2n-1)=n^2$$

$a_n = 2n-1$

$\Rightarrow S_n = n^2$

#1 Show true for $n=1$

$2(1)-1 \stackrel{?}{=} 1^2$

$2-1 = 1$

$1 = 1$ ✓

#2 Assume true for $n=k$

$1+3+5+\ldots+(2k-1)=k^2$

Show true for $n=k+1$ *

$1+3+5+(2k-1)+(2(k+1)-1) = (k+1)^2$

* (from above) $k^2+2k+2-1 \stackrel{?}{=} (k+1)^2$

$k^2+2k+1 \stackrel{?}{=} (k+1)^2$

$(k+1)(k+1) = (k+1)^2$

QED

∴ true for all natural numbers.

Show that the following statement is true for all natural numbers n.

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

#1
Show true for n = 1 : $1 = \frac{1(1+1)}{2}$

$1 = \frac{1(2)}{2}$

#2 $1 = 1$ ✓
assume true for $n = k$:

$$1+2+3+\ldots k = \frac{k(k+1)}{2}$$

Show true for $n = k+1$

$$\underbrace{1+2+3+\ldots k}_{\text{from } n=k \text{ stmnt}} + k+1 = \frac{(k+1)((k+1)+1)}{2}$$

$$\frac{k(k+1)}{2} + k+1 \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$$

(common denom) $\frac{k^2+k}{2} + \frac{(k+1)}{1}\left(\frac{2}{2}\right) \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$

(combine like terms) $\frac{k^2+k+2k+2}{2} \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$

$$\frac{k^2+3k+2}{2} \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$$

(factor) $\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$

QED

∴ true for all natural numbers

Show that the following statement is true for all natural numbers n.

$$1+4+7+\cdots+(3n-2)=\frac{1}{2}n(3n-1)$$

#1 show true for n=1 :

$$3(1)-2=\frac{1}{2}(1)(3(1)-1)$$

$$3-2=\frac{1}{2}(2)$$

$$1=1 \checkmark$$

#2 assume true for n=k

$$1+4+7+\ldots+(3k-2)=\frac{1}{2}k(3k-1)$$

Show true for n=k+1

$$1+\underbrace{4+7+\ldots+(3k-2)}+(3(k+1)-2)\stackrel{?}{=}\frac{1}{2}(k+1)(3(k+1)-1)$$

$$\frac{1}{2}k(3k-1)+3k+3-2\stackrel{?}{=}\frac{1}{2}(k+1)(3k+3-1)$$

$$\frac{3k^2-k}{2}+\frac{(3k+1)}{1}\left(\frac{2}{2}\right)\stackrel{?}{=}\frac{1}{2}(k+1)(3k+2)$$

$$\frac{3k^2-k}{2}+\frac{6k+2}{2}\stackrel{?}{=}\frac{1}{2}(k+1)(3k+2)$$

$$\frac{3k^2-k+6k+2}{2}\stackrel{?}{=}\frac{1}{2}(k+1)(3k+2)$$

$$\frac{3k^2+5k+2}{2}\stackrel{?}{=}\frac{1}{2}(k+1)(3k+2)$$

$$\frac{(k+1)(3k+2)}{2}=\frac{1}{2}(k+1)(3k+2)$$

$\therefore$ true for all natural numbers QED