Precalculus

Lesson 12.2: Arithmetic Sequences

Mrs. Snow, Instructor

When any two numbers in a sequence differ by a constant value, the sequence is identified as an **Arithmetic Sequence.**

An arithmetic sequence may be **defined recursively** as:

$$a_1 = a, \qquad a_n - a_{n-1} = d$$

difference of 2 consecutive terms

For n arithmetic sequence $\{a_n\}$ whose first term is $\boldsymbol{a_1}$ and common difference is ***d***, the ***nth* term** is determined by the formula:

$$a_n = a_{n-1} + d$$

Is 4, 6, 8, 10 .... arithmetic? What is the common difference?

+2 +2 +2 yes, $d = 2$

Determine if the following is an arithmetic sequence, find the first term and the common difference:

$$\{s_n\} = \{3n + 5\}$$

$S_1 = 3+5 = 8$

$S_2 = 3(2)+5 = 11$

$S_3 = 9+5 = 14$

yes arithmetic

$d = 11-8 = \underline{3}$

Determine if the following is an arithmetic sequence, find the first term and the common difference:

$$\{t_n\} = \{4 - n\}$$

$t_1 = 4-1 = 3$

$t_2 = 4-2 = 2$

$t_3 = 4-3 = 1$

yes arithmetic

$2-3 = -1 = \underline{d}$

nth Term of an Arithmetic Sequence

For an arithmetic sequence $\{a_n\}$ whose first term is a_1 and whose common difference is d, the nth term is determined by the formula

$$a_n = a_1 + (n-1)d$$

Find the forty-first term of the arithmetic sequence: 2, 6, 10, 14, 18, ...

$d = 6-2 = 4$

$a_{41} = 2 + (41-1)4$

$= 2 + 40(4)$

$= 2 + 160 = \underline{162}$

The 8th term of an arithmetic sequence is 75, and the 20th term is 39.

a) Find the first term and the common difference

b) Give a recursive formula for the sequence.

c) What is the nth term of the sequence?

System of equations

use definition

$a_8 = 75 = a_1 + (8-1)d = a_1 + 7d$

$a_{20} = 39 = a_1 + (20-1)d = a_1 + 19d$ →

$(a_1 + 7d = 75) \cdot -1$ $\quad -a_1 - 7d = -75$

$a_1 + 19d = 39$ $\quad a_1 + 19d = 39$

$12d = -36$

(a)

$d = -36/12 = -3 = d$

$a_1 + 7(-3) = 75$

$a_1 - 21 = 75$

$a_1 = 96$

(b) Write formula w/ a_1 & d

$a_1 = 96$

$a_n = a_{n-1} - 3$

(c)

$a_n = 96 + (n-1)(-3)$

$= 96 - 3n + 3$ Simplify

$a_n = \underline{99 - 3n}$

The sum of the first n terms of an arithmetic sequence is known as a **Partial Sum of an Arithmetic Sequence**

Let $\{a_n\}$ be an arithmetic sequence with first term a_1 and common difference of d. The sum S_n of the first n terms of $\{a_n\}$ may be found in two ways:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$= \sum_{k=1}^{n} [a_1 + (k-1)d] =$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d] \quad *$$

$$S_n = \frac{n}{2}(a_1 + a_n) \quad *$$

Find the sum S_n of the first n terms of the sequence: $\{a_n\} = \{3n + 5\}$

$a_1 = 3+5 = 8$

$a_2 = 3(2)+5 = 11$ $\rightarrow d = 3$

$\sum_{K=1}^{n} 3k+5$

use formula above↑

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$ (SR) $S_n = \frac{n}{2}(a_1 + a_n)$

$= \frac{n}{2}[2(8) + (n-1)3]$ $= \frac{n}{2}(8 + 3n+5)$

$= \frac{n}{2}[16 + 3n - 3]$ $S_n = \frac{n}{2}(13+3n)$

$S_n = \frac{n}{2}[13+3n]$

Find the sum: $60 + 64 + 68 + 72 + \cdots + 120$

$a_1 = 60$

$a_n = 120$

$d = 4$

$n = ??$ go back to n^{th} term equation

$a_n = a_1 + (n-1)d$

$120 = 60 + (n-1)4$

$120 = 60 + 4n - 4$

$64 = 4n \rightarrow n = 16$

$S_n = \frac{16}{2}(60+120) = 8(180) = 1440 = S_n$