Precalculus
Lesson 12.1: Sequences
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A **sequence** is a function *f* whose domain is the set of positive integers. The values *f*(1), *f*(2), *f*(3), … are called terms.

For Example:

$$-2, \quad 4, \quad 6, \quad 8, \quad \ldots \quad , 2n$$

Since we are talking about a function, we can graph sequences.

Write down the first six terms of the following sequence and graph it.

$$\{a_n\} = \left\{\frac{n-1}{n}\right\}$$

Write down the first six terms of the following sequence and graph it.

$$\{b_n\} = \left\{(-1)^{n+1}\left(\frac{2}{n}\right)\right\}$$

Write down the first six terms of the following sequence and graph it.

$$\{c_n\} = \begin{pmatrix} n \ \text{if } n \text{ is even} \\ \frac{1}{n} \ \text{if } n \text{ is odd} \end{pmatrix}$$

Determining a Sequence from a Pattern

Number the terms and see what happens between each term:

(a) $e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \ldots$

(b) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

(c) $1, 3, 5, 7, \ldots$

(d) $1, 4, 9, 16, 25, \ldots$

(e) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots$

Factorial

A factorial is a non negative integer written with an exclamation mark. If $n \geq 0$ is an integer, the factorial is defined as follows:

$$0! = 1 \qquad and\ 1! = 1$$

$$n! = n(n-1) \cdot \ldots \ldots \cdot 3 \cdot 2 \cdot 1 \qquad for\ n \geq 2$$

and…..

$$n! = n(n-1)!$$

Solve:

$9!$

$\frac{12!}{10!}$

$\frac{3!\,7!}{4!}$

A Sequence Defined by a Recursive Formula

A second way of defining a sequence is to assign a value to the first term and specify the nth term by a formula or equation that involves on or more of the terms preceding it. The sequence is defined **recursively**, and the formula is a **recursive formula**.

Write the first 5 terms of the recursive sequence

$$u_1 = 1,\ \ u_2 = 1,\ \ u_n = u_{n-2} + u_{n+1}$$

Sigma Notation: a short cut notation to indicate the sum of some or all of the terms of a sequence.

Given a sequence

$$a_1,\ a_2, a_3, a_4, \dots a_n.$$

we can write the sum of the first n terms using **summation notation**, or **sigma notation.** The notation derives its name from the Greek Letter **Σ.** This corresponds to our S for "sum." The following notation is used

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots a_n$$

k is called the index of summation, it is basically the starting number for the sequence.

Write out each sum $\sum_{k=1}^{10} \frac{1}{k}$	$\sum_{k=1}^{n} k!$
Express each sum using summation notation $1^2 + 2^2 + 3^2 + \cdots + 9^2$	$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{n-1}}$

The following sums are natural consequences of properties of the real numbers. These are useful for adding terms of a sequence algebraically:

Properties of Sequences

If $\{a_n\}$ and $\{b_n\}$ are two sequences and c is a real number, then:

$$\sum_{k=1}^{n}(ca_k) = ca_1 + ca_2 + \cdots + ca_n = c(a_1 + a_2 + \cdots + a_n) = c\sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

$$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} a_k, \quad \text{where } 0 < j < n$$

The formulas for sums of powers of the first *n* natural numbers are important in calculus:

Formulas for Sums of Sequences

$$\sum_{k=1}^{n} c = \underbrace{c + c + \cdots + c}_{n \text{ terms}} = cn \qquad c \text{ is a real number}$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Find the sums:

$$\sum_{k=1}^{5} (3k)$$

$$\sum_{k=1}^{10} (k^3 + 1)$$

$$\sum_{k=1}^{24} (k^2 - 7k + 2)$$

$$\sum_{k=6}^{20} (4k^2)$$