Precalculus Lesson 12.1: Sequences Mrs. Snow, Instructor

A <u>sequence</u> is a function f whose domain is the set of positive integers. The values f(1), f(2), f(3), ... are called terms.

For Example:

 $-2, 4, 6, 8, \dots, 2n$

Since we are talking about a function, we can graph sequences.

Write down the first six terms of the following sequence and graph it.

$$\{a_n\} = \left\{\frac{n-1}{n}\right\}$$

Write down the first six terms of the following sequence and graph it. $\{b_n\} = \left\{ (-1)^{n+1} \left(\frac{2}{n}\right) \right\}$

Write down the first six terms of the following sequence and graph it.

$$\{c_n\} = \begin{pmatrix} n & if \ n \ is \ even \\ \frac{1}{n} & if \ n \ is \ odd \end{pmatrix}$$

Determining a Sequence from a Pattern

Number the terms and see what happens between each term:

(a)
$$e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \dots$$

(b) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$
(c) $1, 3, 5, 7, \dots$
(d) $1, 4, 9, 16, 25, \dots$
(e) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \dots$

Factorial

A factorial is a non negative integer written with an exclamation mark. If $n \ge 0$ is an integer, the factorial is defined as follows:

0! = 1 and 1! = 1

 $n! = n(n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$ for $n \ge 2$

and.....

$$n! = n(n-1)!$$

Solve: 9!			
$\frac{12!}{10!}$			
$\frac{3!7!}{4!}$			

A Sequence Defined by a Recursive Formula

A second way of defining a sequence is to assign a value to the first term and specify the nth term by a formula or equation that involves on or more of the terms preceding it. The sequence is defined **recursively**, and the formula is a **recursive formula**.

$$u_1 = 1$$
, $u_2 = 1$, $u_n = u_{n-2} + u_{n+1}$

Sigma Notation: a short cut notation to indicate the sum of some or all of the terms of a sequence.

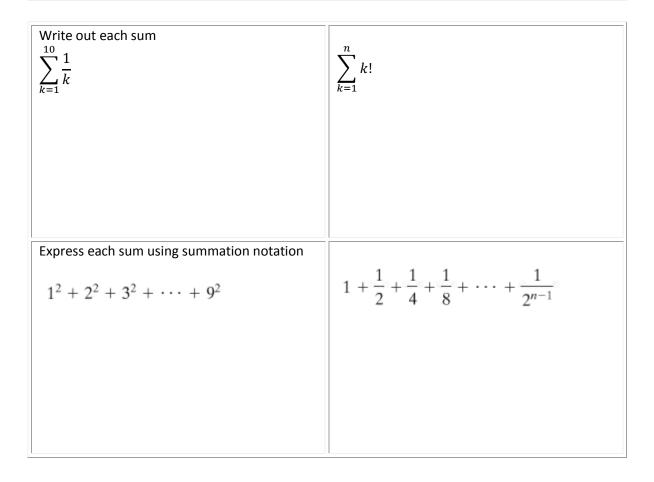
Given a sequence

 $a_1, a_2, a_3, a_4, \ldots a_n.$

we can write the sum of the first n terms using **summation notation**, or **sigma notation**. The notation derives its name from the Greek Letter Σ . This corresponds to our S for "sum." The following notation is used

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots a_n$$

k is called the index of summation, it is basically the starting number for the sequence.



The following sums are natural consequences of properties of the real numbers. These are useful for adding terms of a sequence algebraically:

Properties of Sequences

If $\{a_n\}$ and $\{b_n\}$ are two sequences and c is a real number, then:

$$\sum_{k=1}^{n} (ca_k) = ca_1 + ca_2 + \dots + ca_n = c(a_1 + a_2 + \dots + a_n) = c\sum_{k=1}^{n} a_k$$
$$\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$
$$\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$
$$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} a_k, \text{ where } 0 < j < n$$

The formulas for sums of powers of the first *n* natural numbers are important in calculus: **Formulas for Sums of Sequences**

$$\sum_{i=1}^{n} c = \underbrace{c + c + \dots + c}_{n \text{ terms}} = cn \quad c \text{ is a real number}$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Find the sums:

$$\sum_{k=1}^{5} (3k)$$

$$\sum_{k=1}^{10} (k^3 + 1)$$

$$\sum_{k=1}^{24} (k^2 - 7k + 2)$$

$$\sum_{k=6}^{20} (4k^2)$$