Precalculus
Lesson 10.7: Plane Curves and Parametric Equations
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Think of a point moving in a plane through time. The x- and y- coordinates of the point will then be a function of time. So:

Let $x = f(t)$ *and* $y = g(t)$ where f *and* g are two functions whose common domain is some interval I. The collection of points defined by

$$(x, y) = (f(t), g(t))$$

is called a **plane curve.** The equations

$$x = f(t) \qquad y = g(t)$$

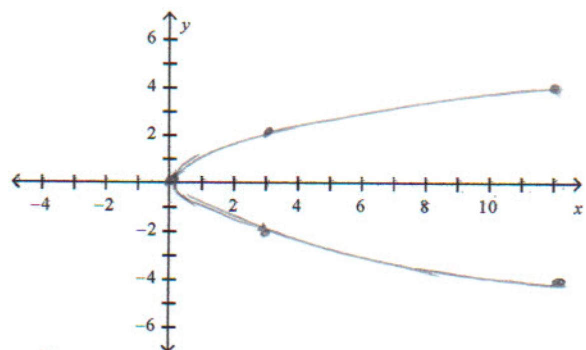
where t is in I are **parametric equations** for the curve. the variable t is called **parmeter.**

Graphing a Curve Defined by Parametric Equations: Notice that for every value of t, we get a point on the curve.

$$x = 3t^2 \qquad y = 2t$$
$$-2 \le t \le 2$$

t	x	y
-2	12	-4
-1	3	-2
0	0	0
1	3	2
2	12	4

graph



Now find the rectangular equation for the parametric curve. combine without "t"

$x = 3t^2 \qquad y = 2t \leftarrow$ solve for t

$x = 3\left(\frac{y}{2}\right)^2 \qquad \frac{y}{2} = t \leftarrow$ substitute in x-t equation

$x = \frac{3y^2}{4} \qquad \leftarrow$ rearrange

$y^2 = \frac{4}{3}x \leftarrow$ parabola opening on positive x-axis vertex at $(0, 0)$

hw: For what values is y defined?

$y = 2t \qquad -2 \le t \le 2 \Rightarrow y = 2(-2) \qquad y = 2(2)$

$y = -4 \qquad y = 4$

$-4 \le y \le 4$

Eliminating the Parameter:

Often a curve given by parametric equations can also be represented by a single rectangular equation in x and y. The process of finding this equation is called eliminating the parameter.

Find the rectangular equation of the curve whose parametric equations are:

$$x = 4\cos t,\ \ and\ \ y = 3\sin t \qquad -0 \le t \le 2\pi$$

$\frac{x}{4} = \cos t \quad \frac{y}{3} = \sin t$ ← Solve for sine and cosine

$\left(\frac{x}{4}\right)^2 = \cos^2 t \quad \left(\frac{y}{3}\right)^2 = \sin^2 t$ ← Square equations

$\cos^2 t + \sin^2 t = 1$ ← Trig Identity

Substitute

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ ← Recognize?

ellipse major axis horizontal

center at origin