Precalculus

Lesson 10.3: The Ellipse

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Ellipse: the collection, or locus, of all points in the plane, the sum of whose distances from two fixed points, called the foci, is constant.

Major axis it's the x-axis:

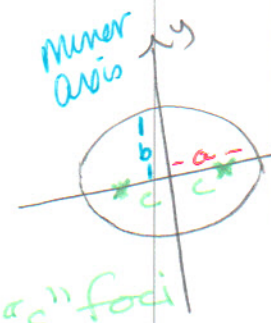
Equation of an Ellipse: Center at (0, 0); Major Axis along the *x*-Axis

An equation of the ellipse with center at $(0, 0)$, foci at $(-c, 0)$ and $(c, 0)$, and vertices at $(-a, 0)$ and $(a, 0)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2 \qquad (2)$$

major axis "a" biggest

biggest number in denominator is the major axis



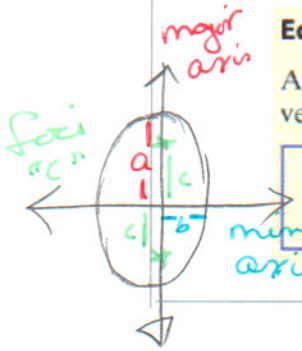
Major axis is the y-axis:

Equation of an Ellipse; Center at (0, 0); Major Axis along the *y*-Axis

An equation of the ellipse with center at $(0, 0)$, foci at $(0, -c)$ and $(0, c)$, and vertices at $(0, -a)$ and $(0, a)$ is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2 \qquad (3)$$

biggest Number => major axis



Finding an equation of an Ellipse

Find an equation of an ellipse with center at the origin, one focus at $(3, 0)$, and a vertex at $(-4, 0)$. Graph.

build equation:

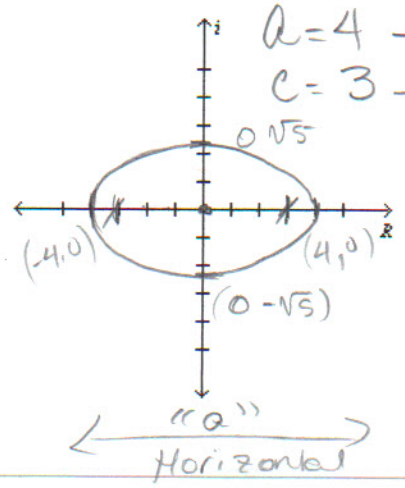
$$\frac{x^2}{16} + \frac{y^2}{5} = 1$$

$a \rightarrow 16$; $5 \leftarrow b^2$

$b^2 = a^2 - c^2$

$b^2 = 16 - 9$

$b^2 = 5 \rightarrow b = \pm\sqrt{5}$



Analyze the Equation of an Ellipse

Analyze (find the center foci and vertices)

$a^2 = 25$ $\quad \frac{x^2}{25} + \frac{y^2}{9} = 1$ $\quad b^2 = 9$

$b = 3$

biggest a

$a = 5$

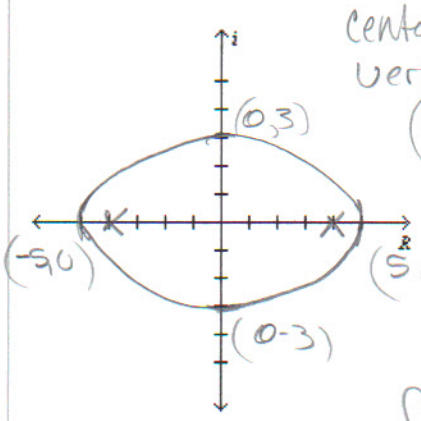
foci

$b^2 = a^2 - c^2$

$9 = 25 - c^2$

$\sqrt{+16} = \sqrt{+c^2}$

$\pm 4 = c$ distance from center to foci



center (0,0)

vertices (5,0) (-5,0)

foci (4,0) (-4,0)

$9x^2 + y^2 = 9$

$\frac{9}{9} \quad \frac{y^2}{9} \quad \frac{9}{9}$

Ellipse must $= 1$ [÷ by 9]

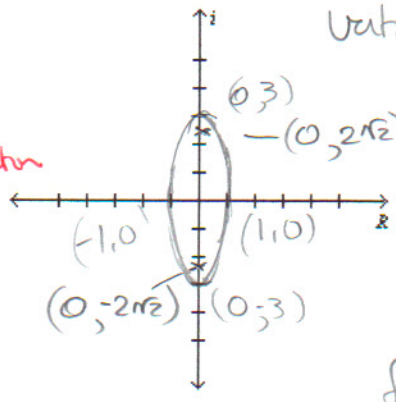
$\frac{x^2}{1} + \frac{y^2}{9} = 1$ ← vertical orientation

$b = 1$ $\quad a = 3$

$9 = 1 - c^2$

$-8 = c^2$

$c = \pm\sqrt{8} = \pm 2\sqrt{2}$



center (0,0)

vertices (0,3) (0-3)

foci $(0, 2\sqrt{2})$ $(0, -2\sqrt{2})$

Find an equation of the ellipse having one focus at $(0, 2)$ and vertices at $(0, -3) and (0, 3)$. Graph.

$c = 2$; $c^2 = 4$

$a = 3$

$a^2 = 9$

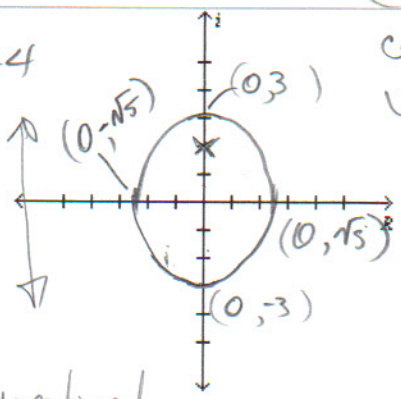
build

b^2? $\frac{x^2}{5} + \frac{y^2}{9} = 1$

$b^2 = 9 - 4$

$b^2 = 5$

$b = \pm\sqrt{5}$



center (0,0)

vertices (0,3) (0,-3)

vertical orientation

foci $(0, \pm 2)$

Ellipses with centers at (h, k)

Equations of an Ellipse: Center at (h, k); Major Axis Parallel to a Coordinate Axis				
Center	**Major Axis**	**Foci**	**Vertices**	**Equation**
(h, k)	Parallel to the x-axis	$(h + c, k)$ $(h - c, k)$	$(h + a, k)$ $(h - a, k)$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$ $a > b > 0$ and $b^2 = a^2 - c^2$
(h, k)	Parallel to the y-axis	$(h, k + c)$ $(h, k - c)$	$(h, k + a)$ $(h, k - a)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,$ $a > b > 0$ and $b^2 = a^2 - c^2$

(h, k)

Find an equation of an ellipse with center at (2, −3), one focus at (3, −3), and one vertex at (5, −3). Graph

$$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{5} = 1$$

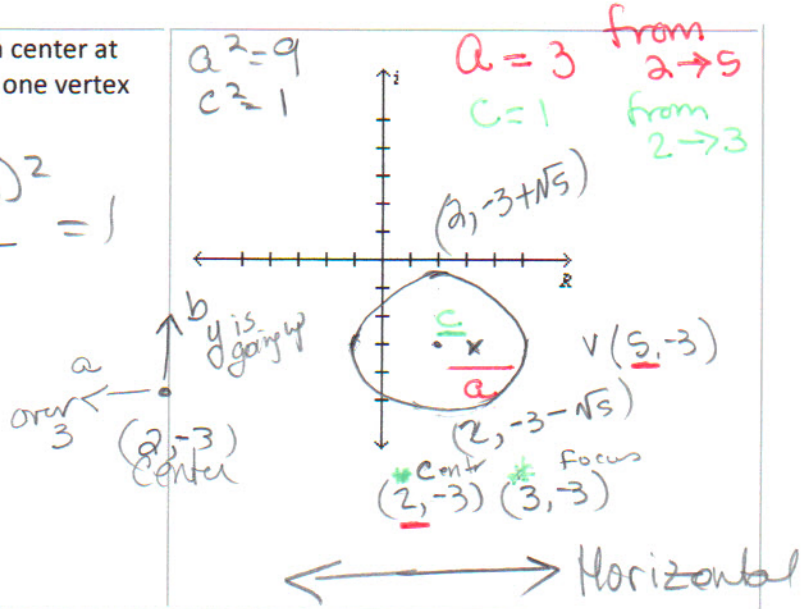
$a^2 = 9$, $a = 3$ (from 2 → 5)

$c^2 = 1$, $c = 1$ (from 2 → 3)

$b^2 = 9 - 1$

$b^2 = 5$

$b = \pm\sqrt{5}$



Analyze the equation:

$$4x^2 + y^2 - 8x + 4y + 4 = 0$$

$4x^2 - 8x + 4 + y^2 + 4y + 4 = 4 + 4$

$4(x^2 - 2x + 1) + (y+2)^2 = 4$

$\frac{4(x-1)^2}{4} + \frac{(y+2)^2}{4} = \frac{4}{4}$

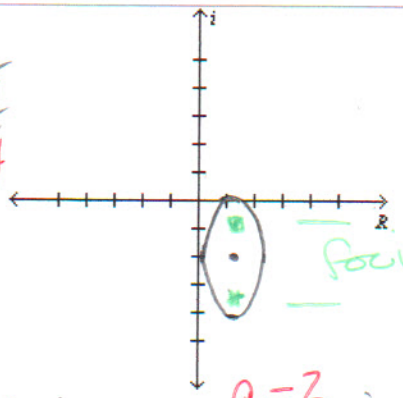
$\frac{(x-1)^2}{1} + \frac{(y+2)^2}{4} = 1$

Vertical Orientation

$1 = 4 - c^2$

$-3 = -c^2$

$c = \pm\sqrt{3}$ = distance from center to foci



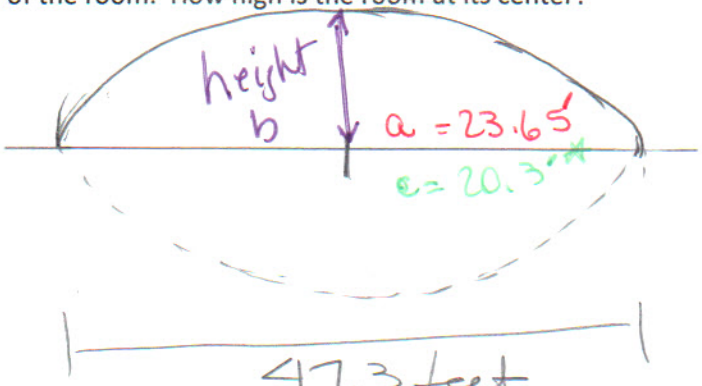
Center (1, −2)

a = 2

Vertices (1, 0) (1, −4)

Foci $(1, -2+\sqrt{3})$ $(1, -2-\sqrt{3})$

The whispering gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center?



center to vertex

$$\frac{x^2}{23.65^2} + \frac{y^2}{147.2325} = 1$$

$b^2 = 23.65^2 - 20.3^2$

$b^2 = 147.2325$

b = height ≈ 12.1 ft