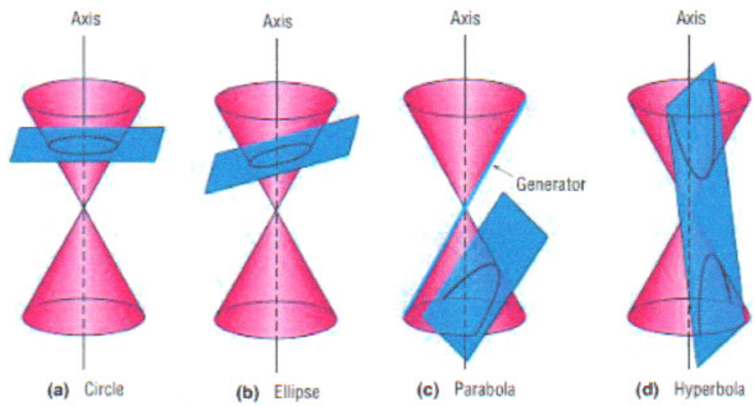
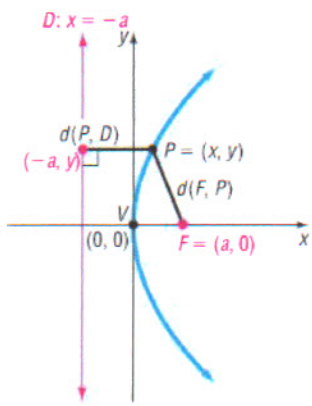
Precalculus
Lesson 10.1 and 10.2: Conics and the Parabola
Mrs. Snow, Instructor

Conic sections are curves that result from the intersection of a cone and a plane. We will be looking at the parabola, ellipse and the hyperbola.



Parabola: A collection, or locus, of all points P in the plane that are the same distance from a fixed point as they are from a fixed line. The point F is the **focus** and the line is its **directrix.**



these distances are equal:

$$d(F,P) = d(P,D)$$

For the parabola that opens along the x-axis:

$$\boldsymbol{y^2 = 4ax}$$ opens x-axis

where:

$\boldsymbol{vertex\ at}\ (\boldsymbol{0},\boldsymbol{0})$, $\boldsymbol{focus\ at}\ (\boldsymbol{a},\boldsymbol{0})$,

"a "is the distance from the vertex to the focus of a parabola

* format different from Alg II

Analyze the equation: $x^2 = -12y$
(find the vertex, focus and directrix and graph)

$x^2 = -12y$, vertex $(0,0)$
focus $(0,-3)$

$-12 = 4a$

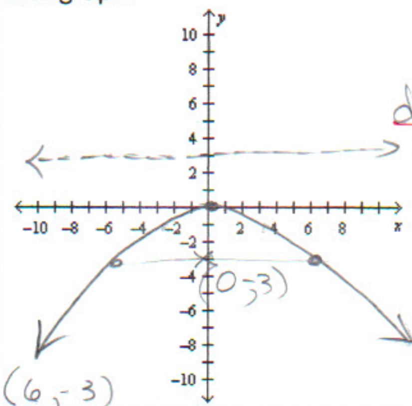
$-3 = a$ to focus neg y

latus rectum points

use $y = -3$ $\quad x^2 = -12(-3)$

$x = \pm 6 \leftarrow x^2 = 36$

and graph:



(6,-3)
(6,3)

Graphing and Finding Equations of Parabolas

Find an equation of a parabola with a vertex at (0,0) and a focus at (3,0). Graph the equation

$a = 3$ opens right

$y^2 = 4ax$

$y^2 = 4(3)x$

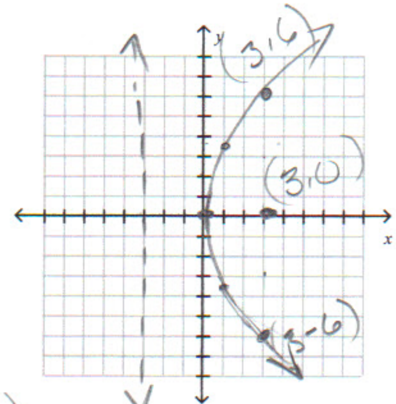
$y^2 = 12x$

(focus) use $x = 3$

$y^2 = 12(3)$

$y^2 = 36$

$y = \pm 6$



(3,6)
(3-6)

$x = -3$

Find an equation of a parabola with a focus at (0,4) and a and directrix line $y = -4$ Graph the equation

opens up

vertex(0,0)

$a = 4$

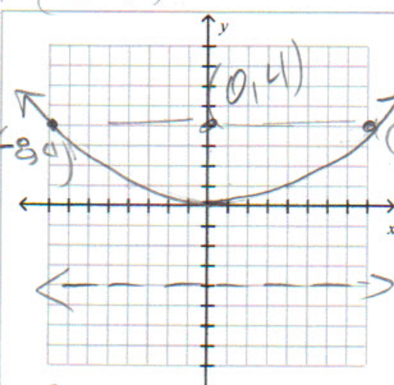
$x^2 = 4(4)y$

$x^2 = 16y$

(focus) use $y = 4$

$x^2 = 64$

$x = \pm 8$



$(\pm 8, 4)$

Find the equation of the parabola with vertex at (0, 0) if its axis of symmetry is the x-axis and its graph contains the point $(-\frac{1}{2}, 2)$

Sketch!!

x y

open left $\Rightarrow$

$y^2 = -4ax$

$2^2 = -4a(-\frac{1}{2})$ sub(x,y)

$4 = -4(-\frac{1}{2})a$

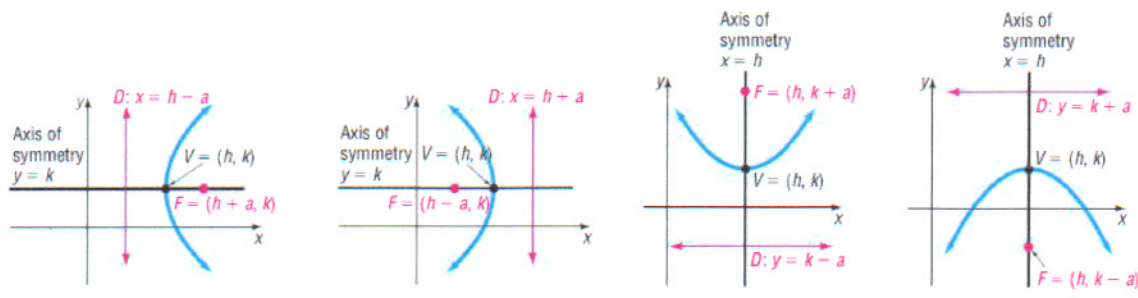
$4 = 2a$

$2 = a$ $\rightarrow$ $y^2 = -8x$

And yes, parabolas may be translated:

Equations of a Parabola; Vertex at (h, k); Axis of Symmetry Parallel to a Coordinate Axis

vertex	focus	directrix	equation	description
(h, k)	$(h+a, k)$	$x = h - a$	$(y-k)^2 = 4a(x-h)$	opens right
(h, k)	$(h-a, k)$	$x = h + a$	$(y-k)^2 = -4a(x-h)$	opens left
(h, k)	$(h, k+a)$	$y = k - a$	$(x-h)^2 = 4a(y-k)$	opens up
(h, k)	$(h, k-a)$	$y = k + a$	$(x-h)^2 = -4a(y-k)$	opens down



(a) $(y-k)^2 = 4a(x-h)$ **(b)** $(y-k)^2 = -4a(x-h)$ **(c)** $(x-h)^2 = 4a(y-k)$ **(d)** $(x-h)^2 = -4a(y-k)$

Finding the Equation of a Parabola, Vertex Not at the Origin

Find an equation of the parabola with vertex at $(-2, 3)$ and focus at $(0, 3)$. Graph.

(h, k) $\quad a = 2$ (from -2 to 0)

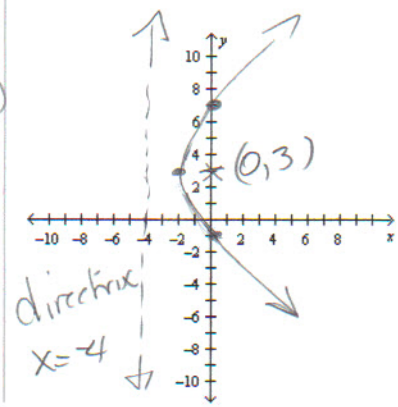
$(y-k)^2 = 4a(x-h)$

$(y-3)^2 = 8(x+2)$

point $\quad x = 0 \leftarrow$ (focus)

$y - 3 = 4 \quad y = 7 \quad (0, 7)$

$y - 3 = -4 \quad y = -1 \quad (0, -1)$



Analyzing the Equation of a Parabola, (find the vertex, focus and directrix and graph)

$x^2 + 4x - 4y = 0$

$x^2 + 4x + 4 = 4y + 4$

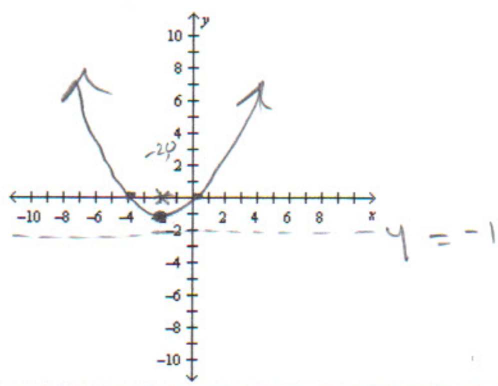
$(x+2)^2 = 4(y+1)$

$(h, k) = (-2, -1)$

$a = 1$ $\quad$ focus $(-2, 0)$

point

$y = 0 \quad (x+2)^2 = 4(1)$



$(x+2)^2 = 4$

$x + 2 = \pm 2$

$x + 2 = 2 \quad x = 0 \quad (0, 0)$

$x + 2 = -2 \quad x = -4 \quad (0, -4)$