## Precalculus

## Lesson 10.4: The Hyperbola

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A hyperbola is the collection (locus) of all points In the plane, the difference of whose distances from two fixed points, called the foci, is a constant.

Equation of a Hyperbola Centered about the origin with Transverse Axis along the $x$-axis

$$
\begin{aligned}
& \frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1 \\
& b^{2}=c^{2}-a^{2}
\end{aligned}
$$

center at $(0,0)$; foci at $( \pm c, 0)$; and vertices at $( \pm a, 0)$
two oblique asymptotes: $y= \pm \frac{b}{a} \boldsymbol{x}$
Find an equation of the hyperbola with center at the origin, one focus at $(3,0)$ and one vertex at $(-2,0)$. Graph


Analyze the equation; find the center, transverse axis, vertices, and foci. Graph. $\frac{x^{2}}{16}-\frac{y^{2}}{4}=1$


## Equation of a Hyperbola; Center at ( 0,0 ); Transverse Axis along the y -axis

$$
\begin{aligned}
& \frac{y^{2}}{a^{2}}-\frac{x^{2}}{b^{2}}=1 \\
& b^{2}=c^{2}-a^{2}
\end{aligned}
$$

center at ( 0,0 ); foci at $(0, \pm \boldsymbol{c})$; and vertices at $(0, \pm \boldsymbol{a})$
two oblique asymptotes: $y= \pm \frac{a}{b} x$

Analyze the equation, find the center, transverse axis, vertices, and foci and graph:

$$
y^{2}-4 x^{2}=4
$$



Find an equation of the hyperbola having one vertex at $(0,2)$ and foci at $(0,-3)$ and $(0,3)$. Graph.


Analyze the equation, find the center, transverse axis, vertices, foci, and asymptotes and graph: $9 x^{2}-4 y^{2}=36$


Hyperbolas at a center of (h,k)

| Opens | Opens left and right <br> Transverse axis <br> x-axis | Opens up and down <br> Transverse axis <br> y-axis |
| :--- | :---: | :---: |
| Form: | $\frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1$ | $\frac{(y-k)^{2}}{a^{2}}-\frac{(x-h)^{2}}{b^{2}}=1$ |
| Center: | $(h, k)$ | $(h, k)$ |
| Vertices | $\pm \frac{b}{a}$ | $(h, k+a) a n d(h, k-a)$ |
| Slope of <br> Asymptotes | $y-k= \pm \frac{b}{a}(x-h)$ | $y-k= \pm \frac{a}{b}(x-h)$ |
| Equation of <br> Asymptotes | $(h+c, k),(h-c, k))$ | $(h, k+c),(h, k-c)$ |
| Foci <br> $a^{2}+b^{2}=c^{2}$ |  |  |

Find an equation for the hyperbola with center at $(1,-2)$, one focus at $(4,-2)$, and one vertex at $(3,-2)$. Graph the equation by hand.



