Precalculus

Lesson 6.6 –Phase Shift; Sinusoidal Curve Fitting

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If we take the transformations from 6.3, we can expand on the transformations to include horizontal and vertical translations.

Amplitude and Period:

$$y = A\sin(\omega x - \phi) + B \qquad\qquad y = A\cos(\omega x - \phi) + B$$

$$y = A\sin\omega\left(x - \frac{\phi}{\omega}\right) + B \longleftrightarrow y = A\cos\omega\left(x - \frac{\phi}{\omega}\right) + B$$

get into this format

Amplitude: $\lvert A\rvert$	Amplitude: $\lvert A\rvert$
Period= $T = \frac{2\pi}{\omega}$	Period= $T = \frac{2\pi}{\omega}$
Phase shift = $\frac{\phi}{\omega}$	*Phase shift* = $\frac{\phi}{\omega}$
B=vertical translation	*B=vertical translation*

The phase shift is to the right if we subtract and the shift is to the left if we have addition.

Find the amplitude, period and phase shift of y, and graph one complete period.

$$y = 3\sin(2x - \pi)$$

$A = 3 \quad \omega = 2 \quad \phi = \pi$

$y = A\sin\omega\left(x - \frac{\phi}{\omega}\right) + B$

$\boxed{y = 3\sin 2\left(x - \frac{\pi}{2}\right)}$

phase shift $\frac{\pi}{2}$ → Rt.

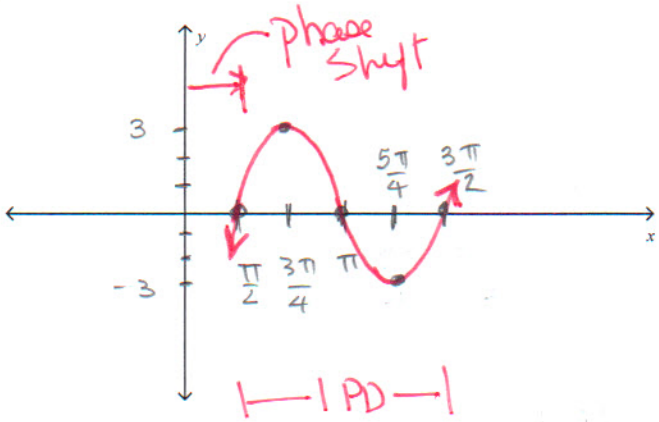
$Pd = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

$Pd \; [0, \pi]$ $+\frac{\pi}{2}$ $+\frac{\pi}{2}$

→ Rt $\frac{\pi}{2}$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

graph x vs y

x	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
$x - \frac{\pi}{2}$	$-\pi/2$ 0	$-\frac{2\pi}{4}$ $\frac{\pi}{4}$	$\frac{\pi}{2}$	$-\frac{2\pi}{4}$ $\frac{3\pi}{4}$	$-\frac{\pi}{2}$ π
$2(x - \frac{\pi}{2})$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin 2(x - \frac{\pi}{2})$	0	1	0	−1	0
$y = 3\sin 2(x - \frac{\pi}{2})$	0	3	0	−3	0



Midpoint Formula $\frac{x_1 + x_2}{2}$

$\frac{\pi}{2} + \pi = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

$\frac{\pi}{2} + \frac{3\pi}{2} = \frac{4\pi}{2} \cdot \frac{1}{2} = \pi$

$\pi + \frac{3\pi}{2} = \frac{5\pi}{2} \cdot \frac{1}{2} =$

$y = A\cos\omega(x - \frac{\phi}{\omega}) + B$

$y = 2\cos(4x + 3\pi) + 1$

$A = 2$ $\omega = 4$ $\phi\ 3\pi$

$B = 1 \uparrow$

$\boxed{y = 2\cos 4(x - \frac{3\pi}{4}) + B}$

Phaseshift $\frac{3\pi}{4}$ ← left

graph

x	$-\frac{3\pi}{4}$	$-\frac{5\pi}{8}$	$-\frac{\pi}{2}$	$-\frac{3\pi}{8}$	$-\frac{\pi}{4}$
$x + \frac{3\pi}{4}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$4(x + \frac{3\pi}{4})$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos 4(x + \frac{3\pi}{4})$	1	0	-1	0	1
$2\cos(\ )$	2 +1	0 +1	-2 +1	0 +1	2 +1
$2\cos(\ ) + 1$	3	1	-1	1	3

$y =$

$Pd = \frac{2\pi}{4} = \frac{\pi}{2}$

Interval

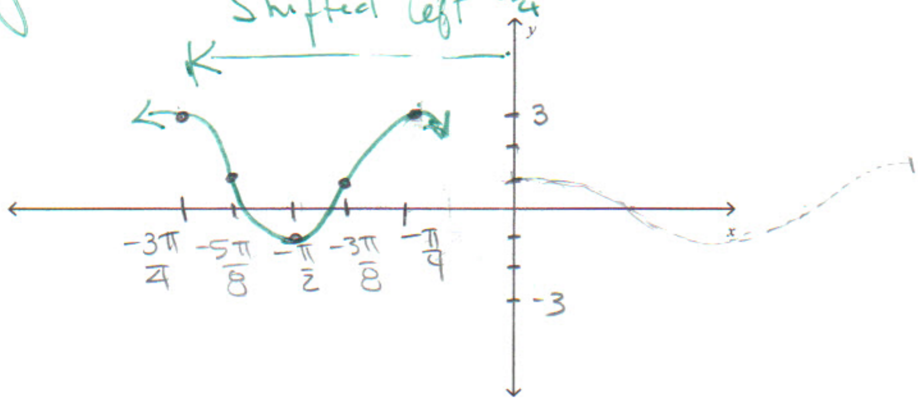
$[0, \frac{\pi}{2}]$ Shift $\frac{3\pi}{4}$

$-\frac{3\pi}{4}$ $-\frac{3\pi}{4}$ ←

$[-\frac{3\pi}{4}, -\frac{\pi}{4}]$

$-\frac{\pi}{4} = \frac{2\pi}{4} - \frac{3\pi}{4}$

Shifted left $\frac{3\pi}{4}$



$\frac{-2\pi}{4} - \frac{\pi}{4} = \frac{-3\pi}{4} \cdot \frac{1}{2}$

$-\frac{3\pi}{4} + \frac{3\pi}{4}$ $\frac{3\pi}{4} - \frac{2\pi}{4}$ $\frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4}$ $-\frac{3\pi}{4} - \frac{\pi}{4} = \frac{-4\pi}{4}\ \frac{1}{2} = \frac{-\pi}{2}$

$-\frac{5\pi}{8} + \frac{6\pi}{8}$ $-\frac{3\pi}{8} + \frac{6\pi}{8}$ $-\frac{6\pi}{8} - \frac{4\pi}{8} = \frac{-10\pi}{8} \cdot \frac{1}{2} = \frac{-5\pi}{8}$

SUMMARY

Steps for Graphing Sinusoidal Functions $y = A\sin(\omega x - \phi) + B$ or $y = A\cos(\omega x - \phi) + B$

STEP 1: Determine the amplitude $|A|$, period $T = \frac{2\pi}{\omega}$, and phase shift $\frac{\phi}{\omega}$.

STEP 2: Determine the starting point of one cycle of the graph, $\frac{\phi}{\omega}$. Determine the ending point of one cycle of the graph, $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$. Divide the interval $\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + \frac{2\pi}{\omega}\right]$ into four subintervals, each of length $\frac{2\pi}{\omega} \div 4$.

STEP 3: Use the endpoints of the subintervals to find the five key points on the graph.

STEP 4: Plot the five key points and connect them with a sinusoidal graph to obtain one cycle of the graph. Extend the graph in each direction to make it complete.

STEP 5: If $B \neq 0$, apply a vertical shift.

For Tangent functions we have a similar form

$$y = A\tan\omega\left(x - \frac{\phi}{\omega}\right) + B \qquad y = A\cot\omega\left(x - \frac{\phi}{\omega}\right) + B$$

$$period = \frac{\pi}{\omega} \qquad phase\ shift = \frac{\phi}{\omega}$$

$$y = \tan 2\left(x - \frac{\pi}{4}\right)$$

$A = 1$ $\omega = 2$

$\frac{\phi}{\omega} = \frac{\pi}{4}$ $\xrightarrow{}$ Rt

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$x-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$2(x-\frac{\pi}{4})$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
tan( )	U	-1	0	1	U

pd $= \frac{\pi}{2}$

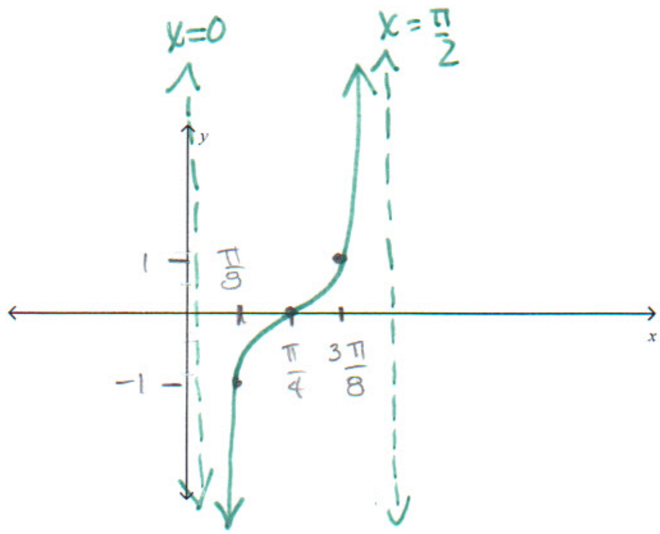
Interval

$\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\rightarrow$ Rt $\frac{\pi}{4}$

$\left(0, \frac{\pi}{2}\right)$

shifted



$\frac{\pi}{8} - \frac{2\pi}{8}$

$\frac{3\pi}{8} - \frac{2\pi}{8}$

$\frac{2\pi}{4} - \frac{\pi}{4}$