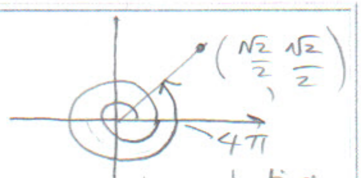
Precalculus
Lesson 6.3: Properties of the Trigonometric Functions
Mrs. Snow, Instructor

Finding the exact value of a trig function and graph

(a) $\sin\frac{17\pi}{4} = \left(\frac{16\pi}{4} + \frac{\pi}{4}\right) =$ 2 revolutions & another $\frac{\pi}{4}$

$\sin\frac{17\pi}{4} = y = \frac{\sqrt{2}}{2}$

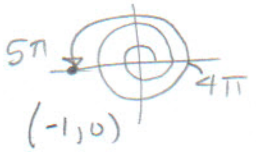


1 revolution = $2\pi\left(\frac{4\pi}{4}\right) = \frac{8\pi}{4}$

(b) $\cos(5\pi)$

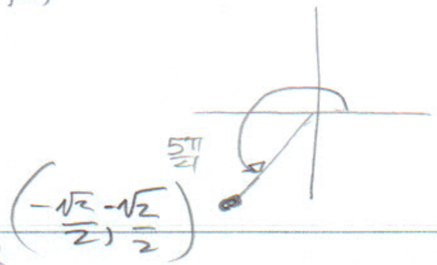
$= \cos(4\pi + \pi)$

$= x = -1$



(c) $\tan\frac{5\pi}{4}$

$= \frac{y}{x} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$



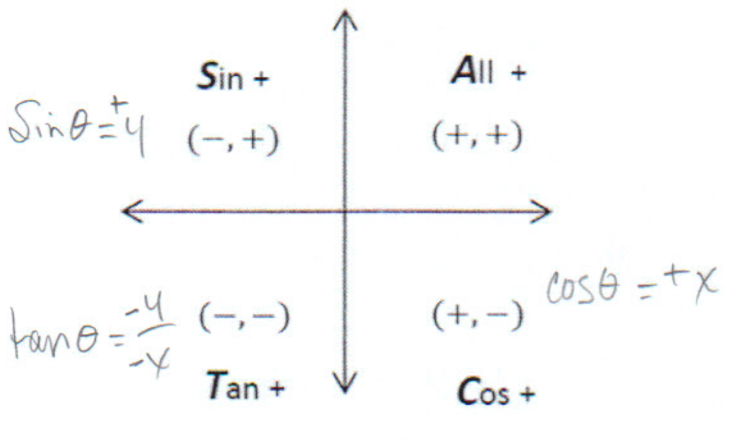
Coterminal: Angles in standard position that share the same terminal side.

SIGNS OF THE TRIGONMETRIC FUNCTIONS

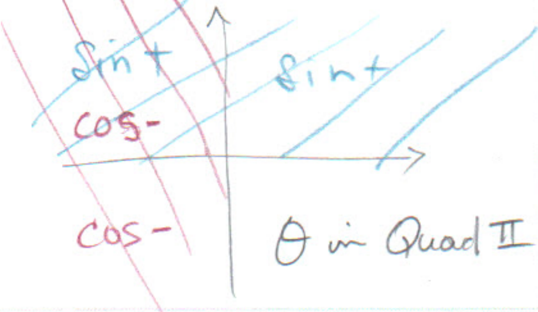
Looking at the Cartesian Plane remember the signs of x and y in each of the quadrants. Superimpose this with the location of **t,** and you can determine the appropriate sign for each trig function.

A little mnemonic to remember which trig function is positive in which quadrant is:

All ***S***tudents ***T***ake ***C***alculus



If $sin\ \theta < 0$ and $cos\ \theta < 0$, name the quadrant in which the angle θ lies.

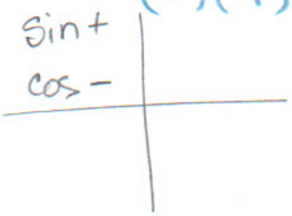


θ in Quad II

Find the sign of the expression if the terminal point is determined by t in the given quadrant.

$$\cos t \cdot \sin t, \quad quadrant\ II$$

$(-)(+) = $ Negative



Fundamental Trigonometric Identities

Trigonometric Identities –In mathematics, trigonometric identities are equalities that involve trigonometric functions and are true for every single value of the occurring variables. The relationship between our basic trig functions and their reciprocals are the **Reciprocal Identities** we also need to know another 3 important identities know as

Reciprocal Identities

$$csc\theta = \frac{1}{sin\theta} \qquad sec\theta = \frac{1}{cos\theta} \qquad cot\theta = \frac{1}{tan\theta}$$

OR

$\sin\theta = \frac{1}{\csc\theta}$

$$tan\theta = \frac{sin\theta}{cos\theta} \qquad cot\theta = \frac{cos\theta}{sin\theta}$$

$\tan\theta = \frac{1}{\cot\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

Pythagorean Identities

$$sin^2\theta + cos^2\theta = 1 \qquad tan^2\theta + 1 = sec^2\theta \qquad 1 + cot^2\theta = csc^2\theta$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Finding exact values using identities

Given $\sin\theta = \frac{\sqrt{5}}{5}$ *and* $\cos\theta = \frac{2\sqrt{5}}{5}$, find the exact values of the four remaining trig functions of θ using identities.

$\sin\theta = \frac{O}{H} = \frac{y}{r} = \frac{\sqrt{5}}{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\cancel{5}\sqrt{5}}{\cancel{5}} = \sqrt{5}$

$\cos\theta = \frac{a}{H} = \frac{x}{r} = \frac{2\sqrt{5}}{5}$

$\sec\theta = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$

$= \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{5}}{\cancel{5}}}{\frac{2\sqrt{5}}{\cancel{5}}} = \frac{1}{2}$

$\cot\theta = \frac{1}{\tan\theta} = 2$

$\cos^2\theta = \frac{1}{\sec^2\theta}$

Find the exact value of each expression. Do not use a calculator

a. $\tan 20° - \frac{\sin 20°}{\cos 20°}$

$\frac{\sin 20°}{\cos 20°} - \frac{\sin 20°}{\cos 20°} = 0$

OR

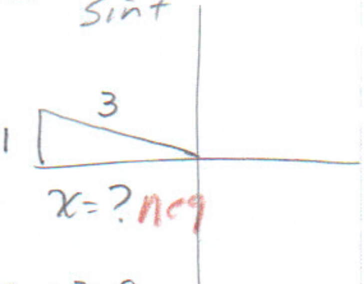
$\tan 20° - \tan 20° = 0$

b. $\sin^2\frac{\pi}{12} + \frac{1}{\sec^2\frac{\pi}{12}}$

$\sin^2\frac{\pi}{12} + \cos^2\frac{\pi}{12} = 1$

Given that $\sin\theta = \frac{1}{3}$ and $\cos\theta < 0$, find the exact value of each of the remaining five trig gunctions.

QII $\cos < 0$
sin +



$1^2 + x^2 = 9$

$x^2 = 8$

$-x = -2\sqrt{2}$

$\sin\theta = \frac{1}{3}$

$\csc\theta = 3$

$\cos\theta = \frac{x}{r} = -\frac{2\sqrt{2}}{3}$

$\sec\theta = \frac{-3}{2\sqrt{2}} = \frac{-3\sqrt{2}}{4}$

$\tan\theta = \frac{\frac{1}{\cancel{3}}}{\frac{-2\sqrt{2}}{\cancel{3}}} = \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$

↑ use for cotangent

$\cot\theta = -2\sqrt{2}$

Given that $tan\theta = \frac{1}{2}$ and $\sin < 0$, find the exact value ~~fo~~ each of the reamaining five trig functions.

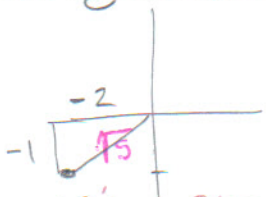
of

+ −

$\sin\theta = \frac{O}{H} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$ $\csc\theta = -\sqrt{5}$

$\cos\theta = \frac{a}{H} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$ $\sec\theta = -\frac{\sqrt{5}}{2}$

$\tan\theta = \frac{O}{a} = \frac{1}{2}$ $\cot\theta = 2$



QIII { sin − , tan + }   sin −

$2^2 + 1^2 = r^2$

$5 = r^2$ $r = \sqrt{5}$

Even-Odd Properties of Trigonometric Functions

A function is considered even if

- $f(-\theta) = f(\theta)$ for all θ in the domain of the function.

The function is odd if

- $f(-\theta) = -f(\theta)$ for all θ in the domain of the function.

Even-Odd Properties

odd $\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$ even	$\tan(-\theta) = -\tan\theta$ odd
odd $\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$ even	$\cot(-\theta) = -\cot\theta$ odd

Find the exact values of:

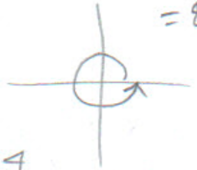
a) $\sin(-45°) = -\sin 45 = -\frac{\sqrt{2}}{2}$

b) $\cos(-\pi) = \cos\pi = -1$

c) $\cot\left(-\frac{3\pi}{2}\right) = -\cot\frac{3\pi}{2} = -\frac{0}{1} = 0$

$\cot\theta = \frac{x}{y}$

$1\text{rev} = 2\pi\left(\frac{4}{4}\right) = \frac{8\pi}{4}$



typo!

d) $\tan\left(-\frac{37\pi}{4\cancel{2}}\right) = -\tan\left(\frac{37\pi}{4}\right) = -\tan\left(\frac{32\pi}{4} + \frac{5\pi}{4}\right)$

$= -\tan\frac{5\pi}{4} = -(1) = -1$

$$\begin{array}{r} 4 \\ 8\overline{)37} \\ 32 \\ \hline 5 \end{array}$$

4 revolutions + 5/8

5 Remainder