Precalculus

Lesson 6.2: Trigonometric Functions: Unit Circle Approach

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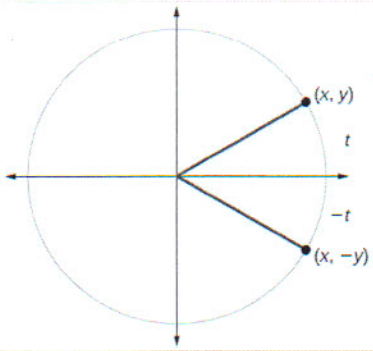
Before we look at the unit circle with respect to the trigonometric functions, we need to get some terminology down for unit circle use. Remember the **Unit Circle has a radius of 1**.

Terminal Point – For and angle in standard position, let $P = (x, y)$ be the point of the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$

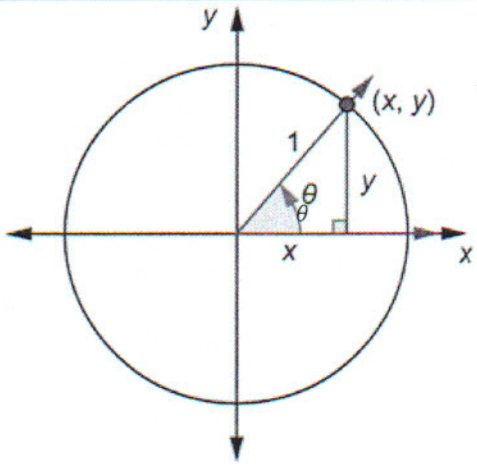


Terminal pt
+
-

Reference angle - The reference angle is always the **smallest** angle that you can make from the terminal side of an angle and the **x-axis**. The reference angle always uses the x-axis as its frame of reference. A reference angle must **be** $< 90°$ **or** $< \frac{\pi}{2} rad.$

135° = θ
θr = 45°

Trig Functions	Reciprocal functions
$\sin t = \frac{opp}{hyp} = \frac{y}{r} = y$	$\csc t = \frac{hyp}{opp} = \frac{r}{y} = \frac{1}{y}$
$\cos t = \frac{adj}{hyp} = \frac{x}{r} = x$	$\sec t = \frac{hyp}{adj} = \frac{r}{x} = \frac{1}{x}$
$\tan t = \frac{opp}{adj} = \frac{y}{x}$	$\cot t = \frac{adj}{opp} = \frac{x}{y}$

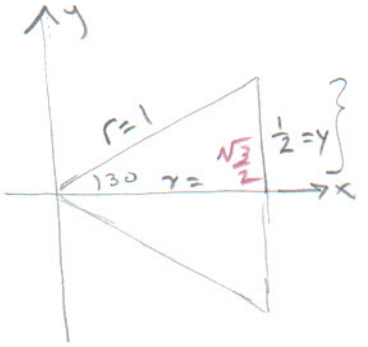


UNIT CIRCLE

The Unit Circle may be constructed using the above idea, a basic understanding of geometry, and recognizing the correlation of the arc distance (terminal point, t) and the degree measure of the angle formed with the radius

SPECIAL RIGHT TRIANGLES.....

Let's review our Special Triangles: $30-60-90$ *and* $45-45-90$



$x^2+y^2=r^2$

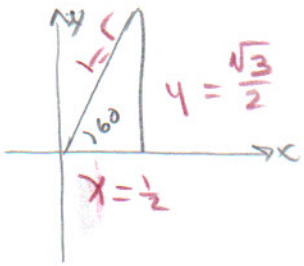
$x^2+\frac{1}{2}^2=1^2$

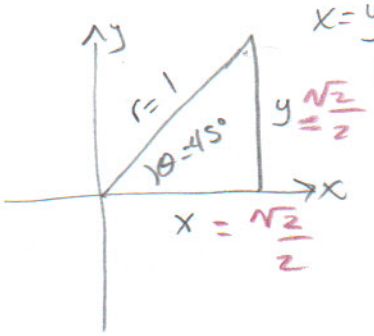
$x^2+\frac{1}{4}=1$

$x^2=3/4$

$x=\frac{\sqrt{3}}{2}$

So.....





$x=y \Rightarrow$

$x^2+x^2=1^2$

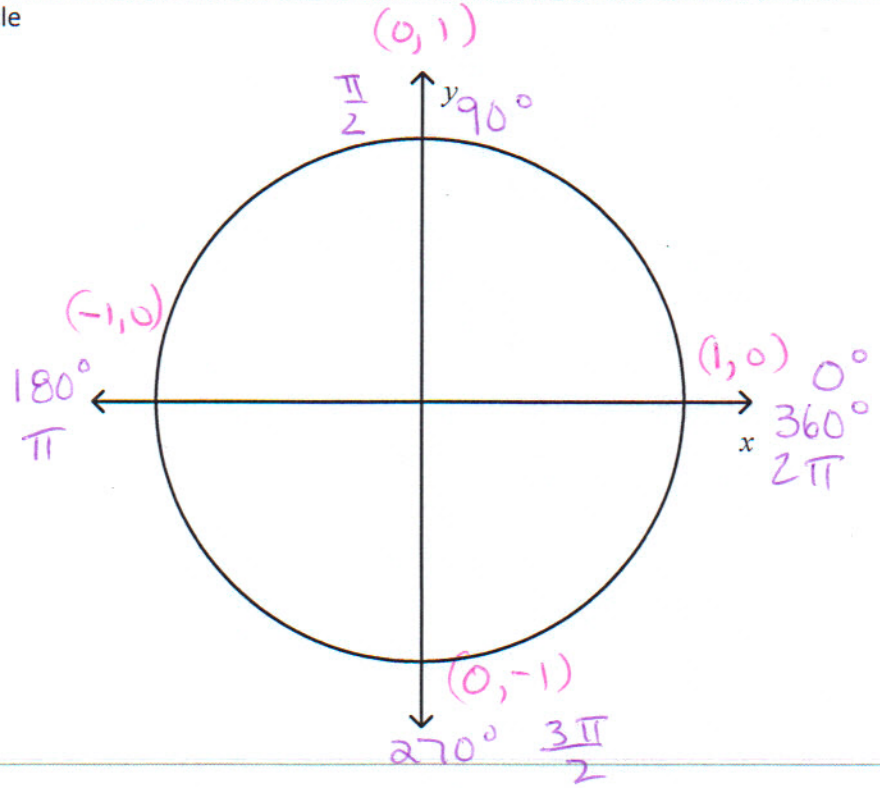
$2x^2=1^2$

$x^2=\frac{1}{2}$

$x=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$

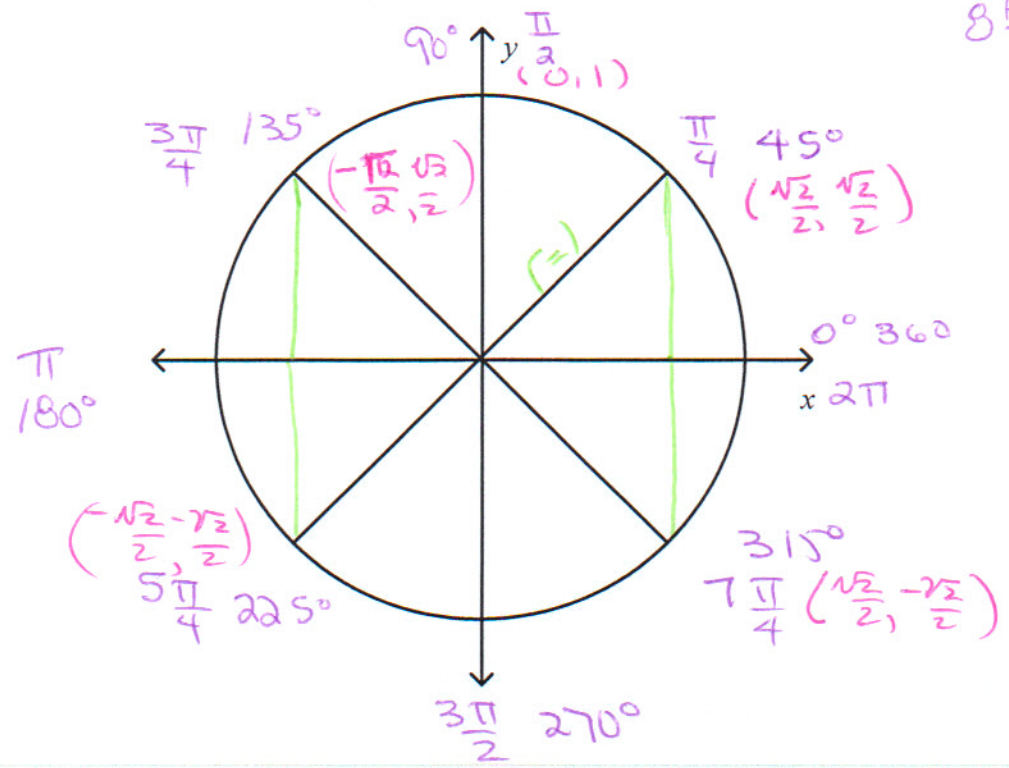
$=\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

The Unit Circle



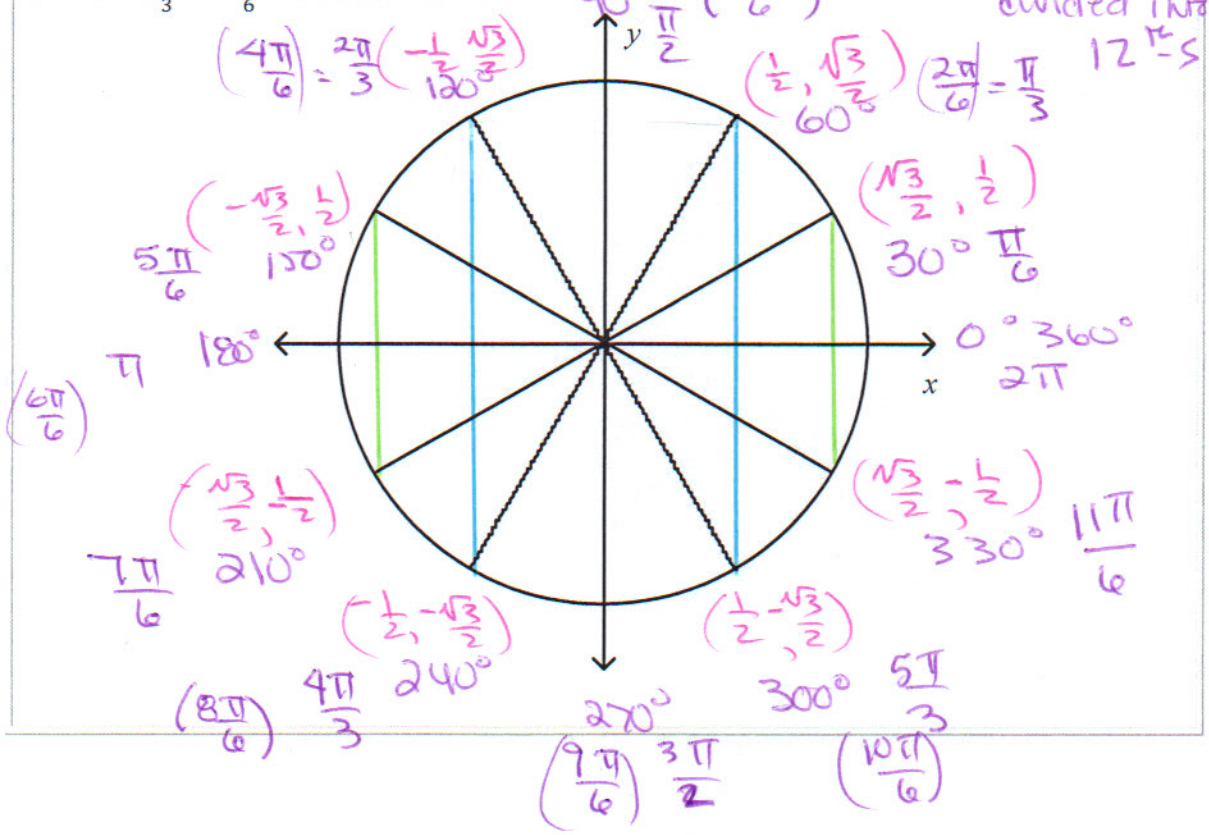
Unit Circle $\frac{\pi}{4}$ reference number

divided into 8 ths

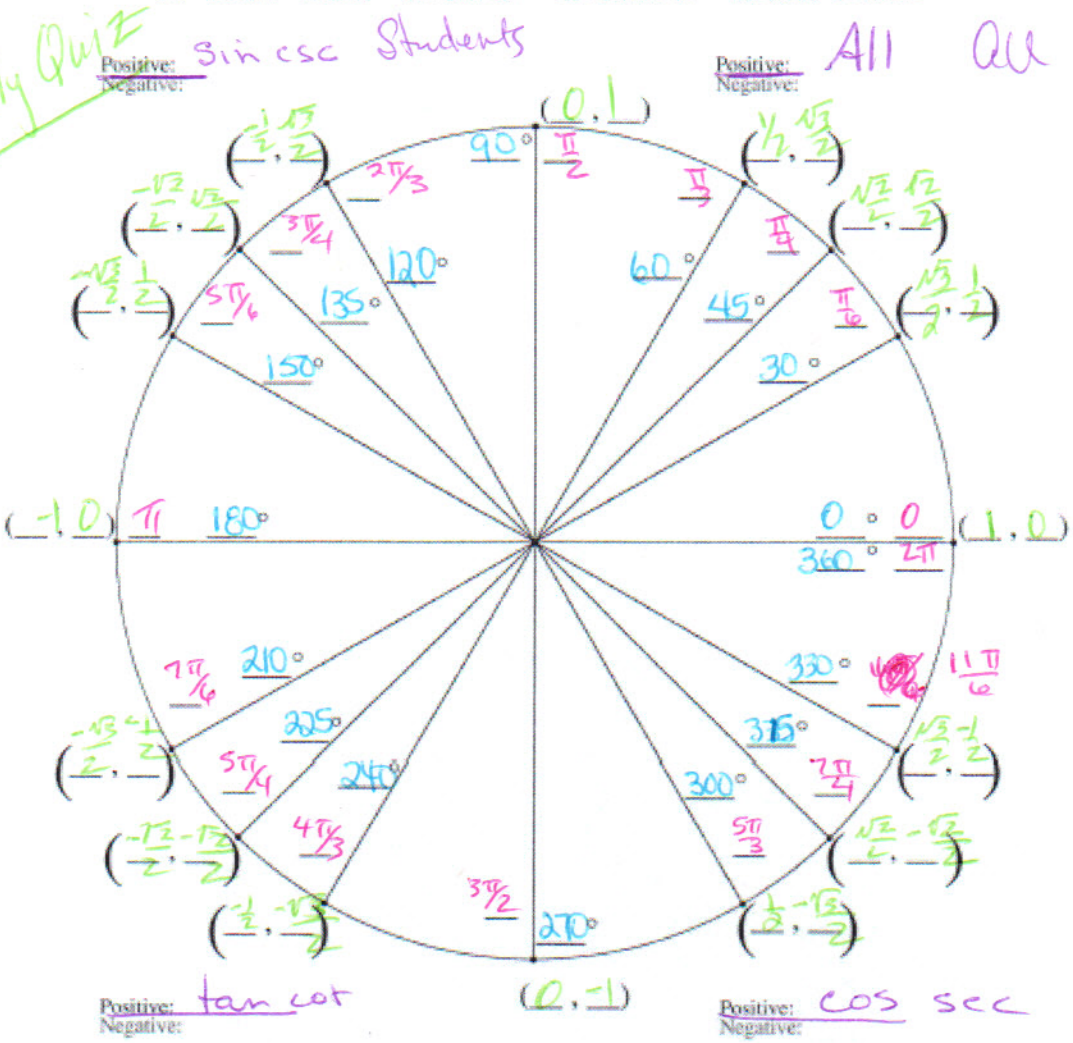


Unit Circle $\frac{\pi}{3}$ *and* $\frac{\pi}{6}$ reference number

divided into 12 ths



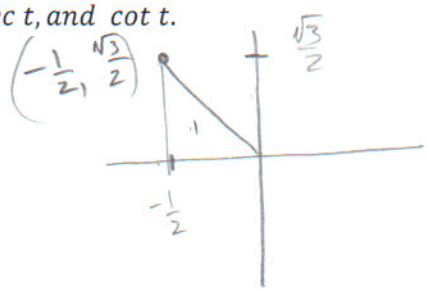
Fill in The Unit Circle



θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Remember Relationship of pagel notes.

Let t be a real number and let $P = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ be the point of the unit circle that corresponds to t Find the values of $\sin t,\ \cos t,\ \tan t,\ \csc t,\ \sec t, \text{and } \cot t.$



$\sin t = \frac{opp}{hyp} = \frac{y}{r} = y = \frac{\sqrt{3}}{2}$

$\cos t = \frac{x}{r} = x = \frac{-1}{2}$

$\tan t = \frac{opp}{adj} = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = -\sqrt{3}$

$\csc t = \frac{1}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec t = \frac{1}{x} = -2$

$\cot = \frac{x}{y} = \frac{\frac{-1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

Finding exact values of the Six Trigonometric Functions using a point on the Unit Circle

Find the exact values of the six trigonometric function of:

a) $\cos\frac{5\pi}{4} = x = \frac{-\sqrt{2}}{2}$ $\sec\theta = -\sqrt{2}$

Unit circle $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\sin\theta = \frac{-\sqrt{2}}{2}$ $\csc\theta = -\sqrt{2}$ $\tan = 1$ $\cot\theta = 1$

b) $\tan 315$

Unit Circle $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$= \tan\frac{7\pi}{4}$

$= \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$ & $\cot\theta = -1$

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\csc\theta = -\sqrt{2}$

$\cos\theta = \sqrt{2}/2$ $\sec\theta = \sqrt{2}$

c) $\sin(-60) = \sin 300°$

U.C $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $= y = -\frac{\sqrt{3}}{2}$

d) $\cos\frac{8\pi}{3} = \cos\left(\frac{6\pi}{3} + \frac{2\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $= x = \frac{-1}{2}$ $\sec\theta = -2$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\csc\theta = \frac{2\sqrt{3}}{3}$

$\tan\theta = -\sqrt{3}$ $\cot\theta = -\frac{\sqrt{3}}{3}$

e) $\csc\frac{\pi}{6} = \frac{r}{y} = \frac{1}{y} = \frac{1}{1/2} = 2$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\sec\theta = \frac{2\sqrt{3}}{3}$

$\tan\theta = \sqrt{3}/3$ $\cot\theta = \sqrt{3}$

f) $\sec 45 = \frac{r}{x} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$ $\sqrt{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $= \sqrt{2}$

Find the exact values a trigonometric function

Find the exact value of each expression

a) $sin45°cos180°$

$\left(\frac{\sqrt{2}}{2}\right)(-1) = -\frac{\sqrt{2}}{2}$

b) $\tan\frac{\pi}{4} - \sin\frac{3\pi}{2}$

$\tan\theta = \frac{y}{x}$ $\sin\theta = y$

$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} - (-1) = 1-(-1)$

$= 2$

c) $(\sec\frac{\pi}{4})^2 + \csc\frac{\pi}{2} =$

$\left(\frac{2}{\sqrt{2}}\right)^2 + 1 =$

$\frac{4}{2} + 1 = 2 + 1 = 3$

$\sec\theta = \frac{1}{\cos\theta}$ & $\csc\theta = \frac{1}{\sin\theta}$

Using a calculator to approximate the value of a trig function: 5decimal places

a) $cos\ 48 \approx .66913$

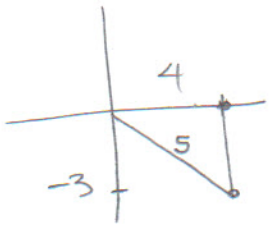
b) $csc\ 21 = 1/\sin(21) \sim 2.79043$

c) $tan\frac{\pi}{12} \sim .26795$

Finding gthe exact value of the six trig functions

Find the exact values of each of the six trig functions of an angle θ *if* $(4, -3)$ is a point on its terminal side in standard position.

$y = -3 \quad x = 4 \quad r = 5$



4, -3

$\sin\theta = \frac{y}{r} = \frac{-3}{5} \qquad \csc\theta = \frac{1}{y} = -\frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5} \qquad \sec\theta = \frac{1}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{-3}{4} \qquad \cot\theta = \frac{x}{y} = \frac{-4}{3}$

$x^2 + y^2 = r^2$

$16 + 9 = 25$

$5 = r$