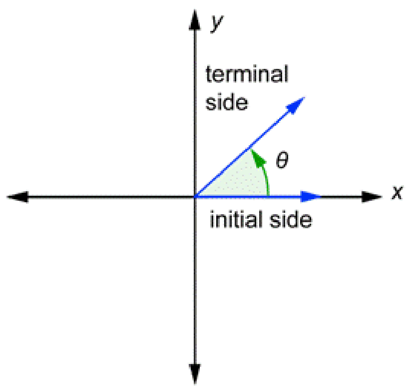
Precalculus
Lesson 6.1: Angles and Their Measure
Mrs. Snow, Instructor

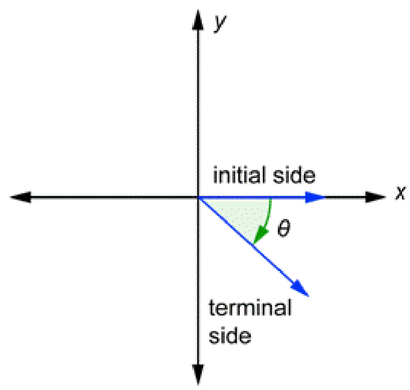
In Trigonometry we will be working with angles from
$0° \ to\ 180° \ to\ 360° \ to\ 720° \ and\ even\ more$!
We will also work with degrees that are smaller than 0°!!

Check out Shaun White's YouTube video of his 2010 Olympic Gold Medal McTwist 1260….yep, it has 1260 degrees of total rotation. Shaun thinks beyond one circle of 360 degrees!

VOCABULARY

Standard Position – An angle is in standard position if its vertex is located at the origin with one ray on the positive x-axis. The ray on the x-axis is called the **initial side** and the other ray is called the **terminal side**.





Positive angle – A positive angle is created by rotating counterclockwise:
$360° = 1$ counterclockwise revolution

Negative angle – A negative angle is measured in the clockwise direction from the positive horizontal axis: $-360°$ =1 clockwise revolution

Draw each angle:

a) $45°$

c) $-90°$

b) $225°$

d) $405°$

Convert between degrees-minutes-seconds (DMS) and decimal measures for angles.

$$1° = 60'$$
$$1' = 60"$$

Calculator: Have the calculator in **degree mode**. For the examples below use the following key strokes. The 2nd apps allows you to select degrees and minutes. The catalog gives you the seconds

[5] [0] [2nd] [APPS] [ENTER] [6] [2nd] [APPS] [▼] [ENTER]
[2] [1] [2nd] [0] [▲] [▲] [▲] [▲] [ENTER] [ENTER]

To convert from decimal degrees for the example given use the following key stokes:

[2] [1] [.] [2] [5] [6] [2nd] [APPS] [ENTER] [2nd]
[APPS] [▼] [▼] [▼] [ENTER] [ENTER]

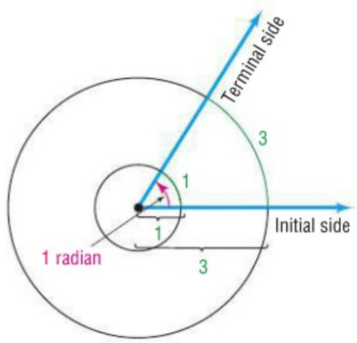
Convert $50°\ 6'\ 21"$ to decimal degrees. Round to four decimal places.

Convert $21.256°$to DMS. Round to the nearest second.

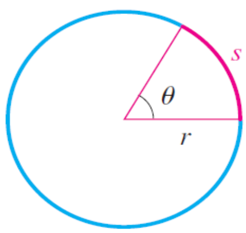
RADIANS

In geometry you studied and learned much about angle measure using degrees. Rotating in a circular path one complete revolution is said to be 360 degrees. Degrees are used to express direction off of magnetic north and angle size. While degree measurements are used in everyday activities such as construction, land surveying, or describing the exact location of a ship at sea or a jet in the sky. Degrees are actually not numbers; they are a fraction of a prescribed total number of degrees to make a complete rotation. Radians are based on the measurement of the circumference of a circle. Radians are used in many science and engineering applications.

If a circle of radius 1 is drawn with the vertex of an angle at its center, then the measure of this angle in radians is the length of the arc that subtends the angle.



LENGTH OF A CIRCULAR ARC



What is the measure of the arc **s** in the diagram to the left? Create a proportion relating the ratio of θ to the whole circle which is 2π *radians* and the ratio of the arc length to the circumference. The proportion simplifies giving the arc length as:

$$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$$

$$\boldsymbol{s = \theta r \quad or \quad \theta = \frac{s}{r}}$$

$$\boldsymbol{\theta\ must\ be\ in\ radians}\text{!!!!!}$$

Finding the length of an arc of a circle

Find the length of the arc of a circle of radius 2 meters subtended by a central angle of 0.25 radian.

If Radius of the circle is 1 unit, what is the circumference of the circle?

$C = 2\pi$r let $r = 1$
$C = 2\pi(1)$
$C = 2\pi$

Conversions between degrees and radians and radians and degrees.

$$\theta = 360° = 2\pi\ rad \ \therefore\ 180° = \pi\ rad$$

$$1° = \frac{\pi}{180} radian \qquad\qquad 1\ rad = \left(\frac{180}{\pi}\right) degrees$$

Converting from degrees to radians

Convert each angle in degrees to radians.

a) $60°$

b) $150°$

c) $-45°$

d) $90°$

Convert each agle in radians to degrees:

a) $\frac{\pi}{6}\ radian$

b) $\frac{3\pi}{2}\ radian$

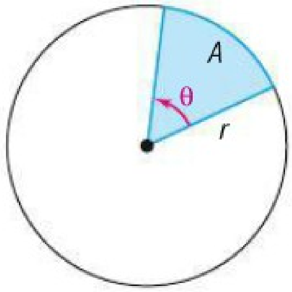
c) $-\frac{3\pi}{4}\ radian$

d) $3\ radians$

Summary (pg. 361):

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

AREA OF A CIRCULAR SECTOR



We can find the area of the "slice of pie" or sector with a central angle θ using proportions. Once again we use the ratio of the angle θ to the whole circle which is $2\pi\ radians$. This is set equal to the ratio of the area of the sector to the area of the circle and simplify.

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

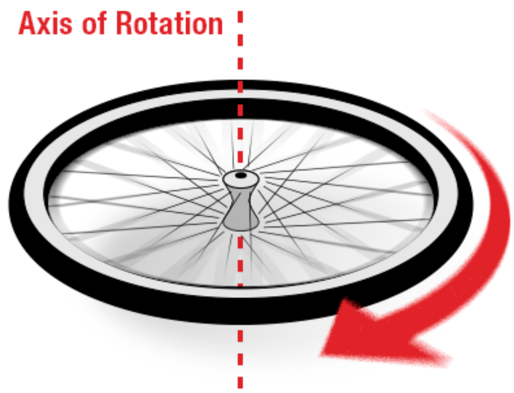
$$\boldsymbol{A = \frac{1}{2} r^2 \theta}$$

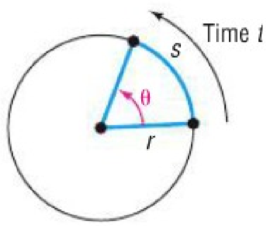
(θ is in radians)

Find the area of the sector of a circle of radius 2 feet formed by an angle of 30°. Round the answer to two decimal places.

CIRCULAR SPEED

When we look at an object moving in a circle, we generally want to know the speed at which it is spinning. **Angular velocity** is the rate at which the object is spinning around its axis. Angular velocity describes **the amount the angle changes** as the object rotates (or revolves) in a specific period of time.





Think about an object spinning in a circular orbit around a point. There are two ways we can describe the speed:

Linear Speed – the rate the distance travelled is changing

Angular Speed – the rate the central angle, θ is changing

Linear Speed (v): linear distance travelled over time.

$$v = \frac{s}{t}$$

- mph (miles per hour)
- Feet per second
- Inches per minute
- Kilometers per hour

Angular Speed (ω): Angle displaced over time.

$$\omega = \frac{\theta}{t}$$

- Degrees per second
- Radians per minute
- Revolutions per minute (RPM)
- Rotations per hour

If a point moves along a circle of radius r with angular speed ω, then its linear speed v is given by:

$$v = r\omega$$

A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.