Interest is the money paid for the use of money. Money borrowed is called **principal.** When you borrow money there is a **rate of interest**, expressed as a percent is charged over the amount of time of the loan. Most often the loan is compounded a number of times per year.

Compound Interest

Compound interest is calculated by the formula:

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

- A(t)= amount after t years
- P=Principal
- r=interest rate per year
- n=number of times compounded per year
- t=number of years

Calculate and compare the amount of money after one year using different compounding periods.

How much money will you have after one year, if you invest $1000 at an annual rate of 10% compounded annually, semiannually, quarterly, monthly, and daily? $r = .1$

$n = 1$ $n = 2$ $n = 4$ $n = 12$ $n = 365$

$A = 1000\left(1 + \frac{.1}{1}\right)^{1}$

$= \$1100$ annual

$A = 1000\left(1 + \frac{.1}{2}\right)^{2}$

$= \$1102.5$ semiannual

$A = 1000\left(1 + \frac{.1}{4}\right)^{4}$

$= \$1103.81$ quarterly

$A = 1000\left(1 + \frac{.1}{12}\right)^{12}$

$= \$1104.71$ Monthly

$A = 1000\left(1 + \frac{.1}{365}\right)^{365}$

$= \$1105.16$ daily

Continuously Compounded Interest

Continuously compounded interest uses the base *e* and is calculated by the formula:

$$A(t) = Pe^{rt}$$

A(t)=amount after t years
P=Principal
r=interest rate per year
t=number of years

Find the amount after 1 year if a principal investment of $1000 is invested at an interest rate of 10% per year, compounded continuously

$$A = 1000 \, e^{(.1)(1)}$$

$1105.17 (a penny more than daily but over many years will add up.)

What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

$$A = 2P$$
$$2P = P\left(1 + \frac{r}{1}\right)^{1(5)}$$
$$\sqrt[5]{2} = \sqrt[5]{(1+r)^5}$$
$$\sqrt[5]{2} = 1+r$$
$$\sqrt[5]{2} - 1 = r = .1487 \implies 14.87\%$$