Precalculus

Lesson 5.6: Logarithmic and Exponential Equations

Mrs. Snow, Instructor

We are now able to apply the previous lessons' formulas to solving logarithmic and exponential equations using algebraic techniques.

Solve:

$2\log_5 x = \log_5 9$

$\log_5 x^2 = \log_5 9$

arguments will be equal

$x^2 = 9$

$x^2 = 3^2$

$x = 3$

$\log_5(x+6) + \log_5(x+2) = 1$

restrictions $\Rightarrow$ $x > -6$ $\quad x > -2$ a.k.a domain

$\log_5 (x+6)(x+2) = 1$

$5^1 = (x+6)(x+2)$

$5 = x^2 + 8x + 12$ $\quad -5$

$0 = x^2 + 8x + 7$

$0 = (x+7)(x+1)$

$x = -7, \; x = -1$

Domain restriction

$\ln x + \ln(x-4) = \ln(x+6)$

$x > 0 \quad x > 4 \quad x > -6$ Domain $(4, \infty)$

$\ln x(x-4) = \ln x + 6$

arguments equal

$x(x-4) = x+6$

$x^2 - 4x - x - 6 = 0$

$x^2 - 5x - 6 = 0$

$(x+1)(x-6) = 0$

$x = -1 \quad x = 6$

Domain restriction

$2^x = 5$

write as a log

$\log_2 5 = x$

$\leftarrow$ change of base

$\frac{\log 5}{\log 2} = x$ (exact solut.)

$2.3219 \sim x$

Exponential equation:

$$8 \cdot 3^x = 5$$

$3^x = \frac{5}{8}$

$\ln 3^x = \ln\left(\frac{5}{8}\right)$

$x \ln 3 = \ln\left(\frac{5}{8}\right)$

$x = \frac{\ln(5/8)}{\ln 3}$ (or)

$x \approx -.428$

$$5^{x-2} = 3^{3x+2}$$

$\ln 5^{x-2} = \ln 3^{3x+2}$

$(x-2)\ln 5 = (3x+2)\ln 3$

$x\ln 5 - 2\ln 5 = 3x\ln 3 + 2\ln 3$

$x\ln 5 - 3x\ln 3 = 2\ln 3 + 2\ln 5$

$x(\ln 5 - 3\ln 3) = 2\ln 3 + 2\ln 5$

$x = \frac{(2\ln 3 + 2\ln 5)}{(\ln 5 - 3\ln 3)}$

$x \approx -3.212$

Quadratic form

$$4^x - 2^x - 12 = 0$$

$4^x = 2^{2x} = (2^x)^2$

So: $(2^x)^2 - 2^x - 12 = 0$

Let $u = 2^x$, we get:

$u^2 - u - 12 = 0$

$(u+3)(u-4) = 0$

replace u with 2^x:

$(2^x+3)(2^x-4) = 0$

$2^x = -3$ or $2^x = 4$

Not possible

$2^x = 2^2$

$\boxed{x = 2}$

Using a calculator:

$$x + e^x = 2$$

$y_1 = x + e^x$

$y_2 = 2$

Intersect at??

$x \approx .44$