Precalculus
Lesson 5.5: Properties of Logarithms
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Logarithms have some very useful properties that can be derived directly from the definition and the laws of exponents:

$$log_a 1 = 0 \qquad a^0 = 1$$

$$log_a a = 1 \qquad a^1 = a$$

Inverse Properties
$$\left\{ \begin{array}{l} a^{log_a M} = M \\ log_a a^r = r \end{array} \right.$$

Simplify:

$2^{log_2 \pi} = \pi$	$log_{0.2} 0.2^{-\sqrt{2}} = -\sqrt{2}$
$ln e^{kt}$ $= log_e e^{kt} = kt$ $[ln\ e = 1]$	$log_4 4 = 1$ dont remember rule: $log_4 4 = x \qquad 4^x = 4^1$ $\therefore x = 1$

Properties of Logarithms

Product property $log_a MN = log_a M + log_a N$

Quotient property $log_a \dfrac{M}{N} = log_a M - log_a N$

Power property $log_a M^r = r\ log_a M$

Write the logarithmic expressions as Sum and Difference Logs expand

$$log_a\left(x\sqrt{x^2+1}\right), x > 0$$

multiply => Add

$= \log_a x + \log_a \sqrt{x^2+1}$

$\sqrt{x} = x^{\frac{1}{2}}$

$= \log_a x + \log_a (x^2+1)^{\frac{1}{2}}$

$= \log_a x + \frac{1}{2}\log_a(x^2+1)$

$$ln\frac{x^2}{(x-1)^3}$$

÷ → subtract

$= \ln x^2 - \ln(x-3)^3$

$= 2\ln x - 3\ln(x-3)$

$$log_a\frac{\sqrt{x^2+1}}{x^3(x+1)^4}$$

÷ subtract

$\log_a \sqrt{x^2+1} - [\log_a x^3(x+1)^4]$

× = add

$\log_a(x^2+1)^{1/2} - [\log_a x^3 + \log_a(x+1)^4]$

$\log_a(x^2+1)^{\frac{1}{2}} - [3\log_a x + 4\log_a(x+1)]$

$= \frac{1}{2}\log_a(x^2+1) - 3\log_a x - 4\log_a(x+1)$

$\ln 1 = 0$

Writing Expressions as a Single Logarithm

$\log_a 7 + 4\log_a 3$ $= \log_a 7 + \log_a 3^4$ $= \log_a 7(3^4)$ $= \log_a 7(81) = \log_a 567$	$\frac{2}{3}\ln 8 - \ln(5^2 - 1)$ $= \ln 8^{2/3} - \ln(25-1)$ $= \ln 2^{3(\frac{2}{3})} - \ln 24$ $= \ln 4 - \ln 24$ $= \ln \frac{4}{24} = \ln \frac{1}{6}$ almost! $= \cancel{\ln 1}^{0} - \ln 6$ $= -\ln 6$

$\log_a x + \log_a 9 + \log_a(x^2 + 1) - \log_a 5$

$= \log_a\left[\frac{(x)(9)(x^2+1)}{5}\right] = \log_a\left[\frac{9x(x^2+1)}{5}\right]$

Change of Base

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

Therefore:

base goes to "basement"

$$\log_a M = \frac{\log M}{\log a} \quad \text{and} \quad \log_a M = \frac{\ln M}{\ln a}$$

Using the Change of Base Formula

$\log_5 89$ ← Base in Basement $= \frac{\log 89}{\log 5} \sim 2.7889$ ↕ same :) or $\frac{\ln 89}{\ln 5} = 2.7189$	$\log_{\sqrt{2}}\sqrt{5}$ $= \frac{\log\sqrt{5}}{\log\sqrt{2}}$ ~ 2.3219

$\boxed{\text{Also}}$ Find exact value: $\log_2 5 \cdot \log_5 16 = \cancel{\log_2 5} \cdot \frac{4}{\cancel{\log_2 5}}$ $\boxed{= 4}$

$\log_5 16 = \frac{\log_2 16}{\log_2 5} = \frac{\log_2 2^4}{\log_2 5} = \frac{4}{\log_2 5}$

↑

Change to base: base 2 like 1st term