Precalculus
Lesson5.3: Exponential Equations
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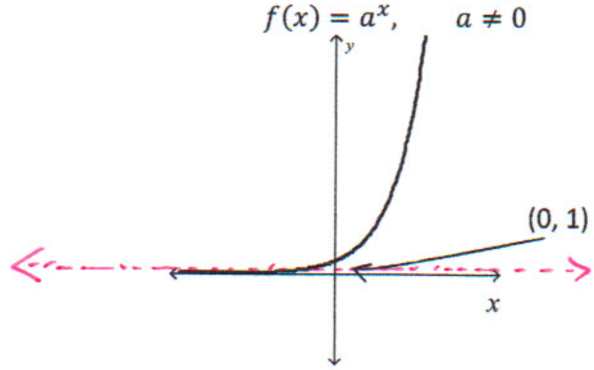
The exponential function is one of the most important functions in mathematics. The function is used to model the natural process of population growth and radioactive decay. It is also important in finances such interest and depreciation. The exponential function with base ***a*** is defined for all real numbers ***x*** by:

$$\boldsymbol{f(x) = Ca^x}$$
$$\boldsymbol{y = Ca^x}$$
$$\boldsymbol{where\ a > 0\ and\ a \neq 1}$$
a is the growth factor
C is the initial value

Graphs in the form of $f(x) = a^x$ will pass through a common point of (0, 1) because $a^0 = 1$ (when $a \neq 0$). The domain of this function will be all real numbers and the range is values greater than 0. What is the horizontal line that y-values will never cross called?????

$f(x) = a^x, \quad a \neq 0$



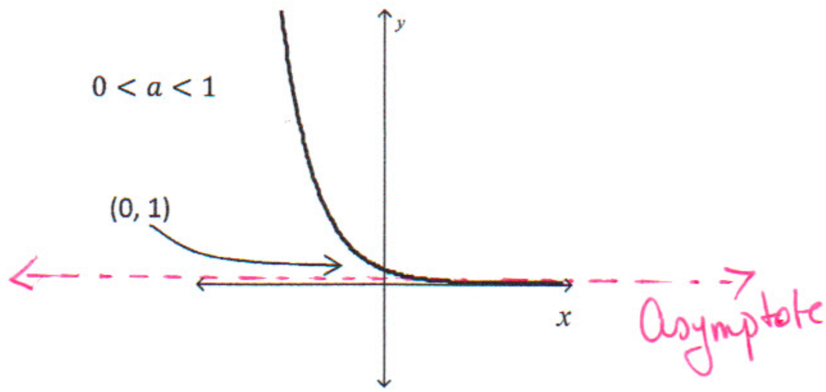
critical pt.
$a^0 = 1$

asymptote
at $y = 0$

Properties of the Exponential Function $f(x) = a^x, a > 1$

1. The domain is the set of all real numbers or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers or $(0, \infty)$ using interval notation.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis $(y = 0)$ is a horizontal asymptote as $x \to -\infty$ $\left[\lim_{x \to -\infty} a^x = 0\right]$.
4. $f(x) = a^x$, where $a > 1$, is an increasing function and is one-to-one.
5. The graph of f contains the points $(0, 1)$, $(1, a)$, and $\left(-1, \frac{1}{a}\right)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

$f(x) = a^x, \quad a \neq 0$

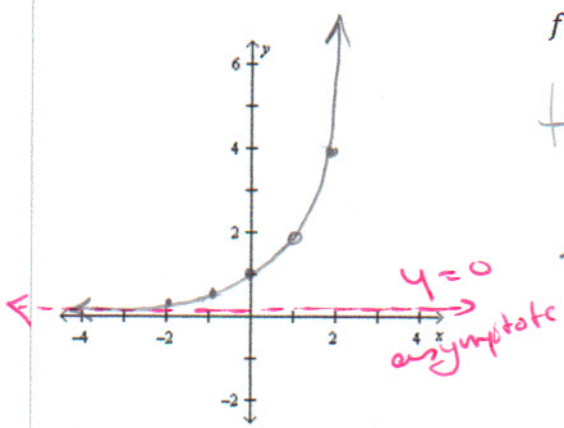


Properties of the Exponential Function $f(x) = a^x, 0 < a < 1$

1. The domain is the set of all real numbers or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers or $(0, \infty)$ using interval notation.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis ($y = 0$) is a horizontal asymptote as $x \to \infty$ $\left[\lim_{x\to\infty} a^x = 0\right]$.
4. $f(x) = a^x$, $0 < a < 1$, is a decreasing function and is one-to-one.
5. The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

Graph the Exponential Functions

$f(x) = (2)^x$



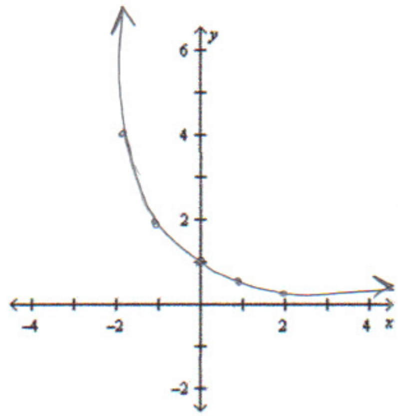
Range $(0, \infty)$

Domain $(-\infty, \infty)$

table of values

x	y
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

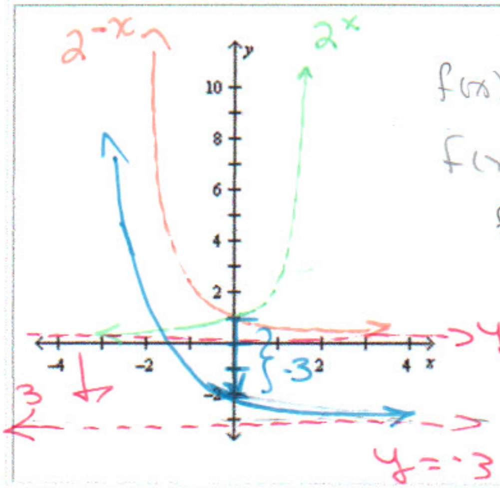
$$f(x) = \left(\frac{1}{2}\right)^x$$



x	y
-2	$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\frac{1}{2}^0 = 1$
1	$\frac{1}{2}^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Graphing Exponential Functions Using Transformations

$$f(x) = 2^{-x} - 3$$



$f(x) = 2^x$

$f(x) = 2^{-x}$ reflects about y-axis

$f(x) = 2^{-x} - 3$ vertical slide down 3 ↓

y-intercept is critical point

asymptote ↓ 3 units

The Natural Exponential Number

A chap named Leonard Euler named this irrational number $e = 2.71828$ .... Applications include the naturally occurring processes of continuous growth and decay and may also be used to model any growth/decay that is continuous.

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The ***number e*** is defined as the number the number in the expression:

$$\left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{n}}\right)^{\boldsymbol{n}}$$

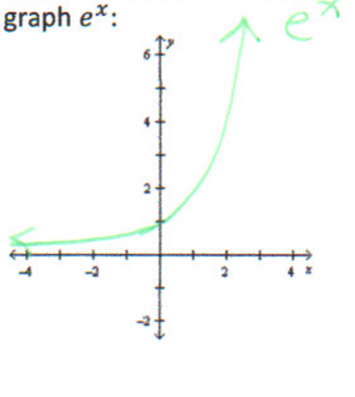
The main thing we need to recognize that and exponential equation my be expressed with a base value of e"

$$\boldsymbol{y = e^x}$$

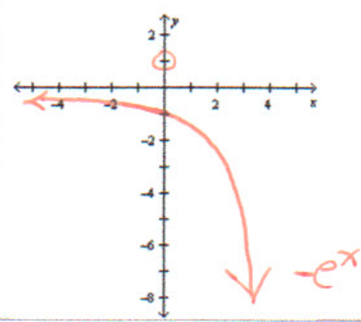
Sketch the graph of the function:

$$f(x) = -e^{x-3}$$

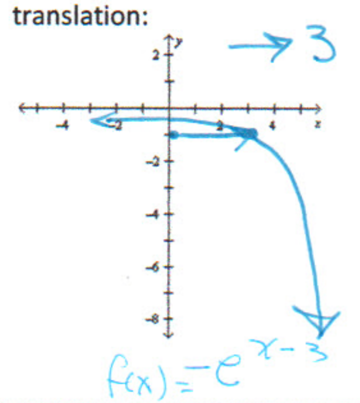
graph e^x:



reflection:



translation:



Solve an Exponential Equation:

$$4^{2x-1} = 8^x$$

If bases are equal then exponents are equal

so:

$4 = 2^2 \qquad 8 = 2^3$

$4^{2x-1} = 8^x$

$2^{(2)(2x-1)} = 2^{3x}$

$2(2x-1) = 3x$

$4x - 2 = 3x$

$\boxed{x = 2}$

$$e^{-x^2} = (e^x)^2 \frac{1}{e^3}$$

$e^{-x^2} = \dfrac{e^{2x}}{e^3}$

$e^{\boxed{-x^2}} = e^{\boxed{2x-3}}$

$-x^2 = 2x - 3$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\boxed{x = -3, \ x = 1}$