Precalculus

Lesson 5.5: Properties of Logarithms

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Logarithms have some very useful properties that can be derived directly from the definition and the laws of exponents:

$$log_a 1 = 0$$

$$log_a a = 1$$

$$a^{log_a M} = M$$

$$log_a a^r = r$$

Simplify:

$2^{log_2 \pi}$	$log_{0.2} 0.2^{-\sqrt{2}}$
$ln e^{kt}$	$log_4 4$

Properties of Logarithms

$$log_a MN = log_a M + log_a N$$

$$log_a \frac{M}{N} = log_a M - log_a N$$

$$log_a M^r = r\, log_a M$$

Write the logarithmic expressions as Sum and Difference Logs

$log_a\left(x\sqrt{x^2+1}\right), x > 0$
$ln\frac{x^2}{(x-1)^3}$
$log_a\frac{\sqrt{x^2+1}}{x^3(x+1)^4}$

Writing Expressions as a Single Logarithm

$log_a 7 + 4log_a 3$	$\frac{2}{3}\ln 8 - \ln(5^2 - 1)$
$log_a x + log_a 9 + log_a(x^2 + 1) - log_a 5$	

Change of Base

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

Therefore:

$$\log_a M = \frac{\log M}{\log a} \quad \text{and} \quad \log_a M = \frac{\ln M}{\ln a}$$

Using the Change of Base Formula

$log_5 89$	$log_{\sqrt{2}}\sqrt{5}$