Precalculus

Lesson 5.2: One to One Functions; Inverse Functions

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A quick definition review: A function is a special relation for which every element of the domain corresponds to exactly one element of the range. Now, for a function to be considered **one-to-one or it may be written as "1-1,"** it must also meet the following criteria: every element of the range corresponds to exactly one element of the domain. Another way to look at this is that each ***x*** in the domain has one and only one corresponding point in the range.

A function is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. That is

$$f(x_1) \neq f(x_2) \quad for\ x_1 \neq x_2$$

Graphically we can determine a one-to-one relationship by using the **horizontal-line-test** to determine of *f* is one-to-one. Basically, this is analogous to the vertical line test, only horizontal.

Horizontal-line Test

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Using the graph of the function to determine if the functions are 1-1

(a) $f(x) = x^2$ (b) $g(x) = x^3$

Inverses: Another way of saying inverse is opposite. Did you ever play "opposite day" with your parents? If you remember you probably would not confess it; Yes means No and No means Yes! Mathematically: $x\ is\ y\ and\ y\ is\ x$.

$f(x)\ and\ g(x)$ are inverses if and only if:

$$f(g(x)) = x \qquad and \qquad g(f(x)) = x$$

Suppose that f is a one-to-one function. Then, to each x in the domain of f, there is exactly one y in the range (because f is a function); and to each y in the range of f, there is exactly one x in the domain (because f is one-to-one). The correspondence from the range of f back to the domain of f is called the **inverse function of *f*.** The symbol f^{-1} is used to denote the inverse of f.

Find the inverse of the following one-to-one function:

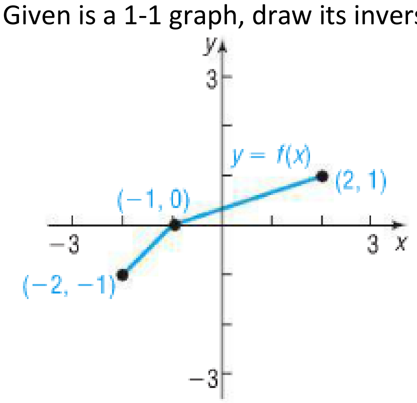
$$\{(-3,-27),(-2,-8),(-1,-1),(0,0),(1,1),(2,8),(3,27)\}$$

(a) Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$.

(b) Verify that the inverse of $f(x) = 2x + 3$ is $f^{-1}(x) = \frac{1}{2}(x - 3)$.

Verify that the inverse of $f(x) = \dfrac{1}{x-1}$ is $f^{-1}(x) = \dfrac{1}{x} + 1$. For what values of x is $f^{-1}(f(x)) = x$? For what values of x is $f(f^{-1}(x)) = x$?

Given is a 1-1 graph, draw its inverse



Find the inverse of $f(x) = 2x + 3$. Graph f and f^{-1} on the same coordinate axes.

The following function is one-to-one. Find its inverse and check the result.

$$f(x) = \frac{2x+1}{x-1}, x \neq 1$$

By restricting the domain of a function that is not 1-1, we can make the function 1-1 and find its inverse.

Find the inverse of $y = f(x) = x^2$ if $x \geq 0$. Graph f and f^{-1}.