Precalculus

Lesson 4.6: Polynomial and Rational Inequalities

Mrs. Snow, Instructor

This section covers the processes to graph inequalities of polynomials of a 3rd degree or greater. Graphically, we are able to visually see where the function is greater or less than 0. However, we also need to demonstrate how to solve an inequality algebraically.

Solve inequality graphically:

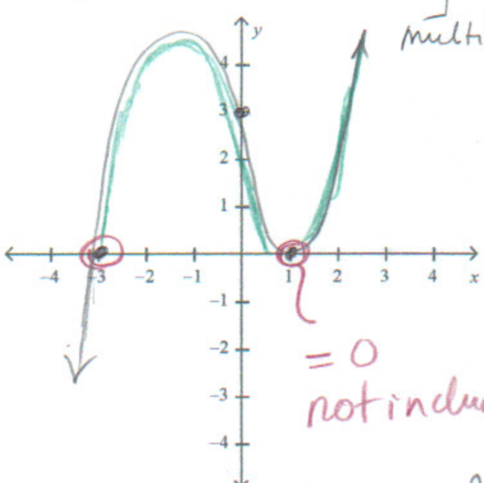
1. Graph and identify the zeros of the function written as an equality.
2. For rational functions, determine the intervals of x such that the graph is above/below the x-axis
3. From the graph you can see where the $f(x)\ is > or < 0$

Sketch!

$(x+3)(x-1)^2 > 0$ cubi

multip = 1 multiplicity = 2

y-int = 3(-1)(-1)
= 3



=0
not included

where is the output greater than zero?

Interval f(x) > 0

$(-3, 1) \cup (1, \infty)$ or

$\{x \mid -3 < x < 1 \text{ or } x > 1\}$

Algebraic solution

1. Write the inequality so that a polynomial/rational expression is on the left side and 0 is on the right side
2. Determine the real zeros (x-intercepts) of *f*. Rational: real numbers for which the expression is undefined.
3. Using the zeros divide the real number line into intervals
   a. Is the inequality $<, >, \leq, or \geq$ at zero?
   b. Equality means a point on the zero
   c. No equal means a circle
4. Select a number in each interval, evaluate at that number. Focus on the sign of the factors and the overall outcome of $\pm$. Don't worry about the exact numerical answer.

Solve the inequalities algebraically and graph the solution

$x^4 > x$ $\quad x^4 - x > 0 \leftarrow f(x) > 0$ not equal to end points

- solve as equality to find zeros; these are end points on solution interval(s)

$x^4 - x = 0$

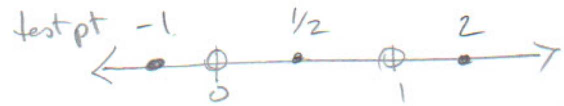
$x(x^3 - 1) = 0$

$x(x-1)(x^2+x+1) = 0$

$x = 0 \qquad x - 1 = 0 \qquad x^2 + x + 1 = 0$*

$\qquad\qquad x = 1 \qquad$ no real solutions

discriminant:
*$b^2 - 4ac$
$1 - 4(1)(1) = -3$



- number line sets up intervals of possible solutions

x	$-$	$+$	$+$
$x^3 - 1$	$-$	$-$	$+$
$x(x^3-1)$	$+$	$-$	$+$

Intervals

$(-\infty, 0) \cup (1, \infty)$

$\{x \mid x < 0 \text{ or } x > 1\}$

- Choose test points in intervals evaluate function factors for +/- and then sign of function

$$\frac{4x+5}{x+2} \geq 3$$

$$\frac{4x+5}{x+2} - 3 \geq 0$$ → positive

common denominator

$$\frac{4x+5}{x+2} - \frac{3(x+2)}{x+2} \geq 0$$

$$\frac{4x+5-3(x+2)}{x+2} \geq 0$$ $4x+5-3x-6$

$$\frac{x-1}{x+2} \geq 0$$

Find zeros at $y=0$: $\frac{x-1}{x+2} = 0$

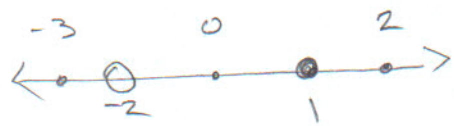
$x-1=0$

$\underline{\underline{x=1}}$

denominator undefined at $\underline{\underline{x=-2}}$

Intervals:

Test pt



$x-1$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
$\frac{x-1}{x+2}$	$+$	$-$	$+$

$$(-\infty, -2) \cup [1, \infty)$$