

**Precalculus**  
**Lesson4.5: The Graph of a Rational Function**  
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Calculators of course make graphing rational function much easier and quicker. However, we need to be proficient in using algebraic analysis to draw conclusions of the graph.

Analyze the rational function:

$$R(x) = \frac{x - 1}{x^2 - 4}$$

1. Factor the numerator and denominator
2. Write R in lowest terms, factor the numerator and denominator
3. Locate the intercepts of the graph:
  - a. x-intercepts: determine the real zeros of the numerator
  - b. y=intercepts: solve for  $R(0)$
4. Locate the vertical asymptotes: find the zeros of the denominator
5. Locate the horizontal/oblique asymptotes: determine whether the function is proper or improper and follow the processes presented in lesson 4.4. Determine the points, if any, at which the graph of R intersects this asymptote. Graph the asymptotes using a dashed line. Plot any points at which the graph of R intersects the asymptote.
6. Graph R using a graphing calculator. And use the results from the analyses to graph R by hand.

Analyze the rational function:

$$\frac{x^2 - 1}{x}$$

Analyze the rational function:

$$\frac{x^4 + 1}{x^2}$$

Analyze the rational function:

$$\frac{3x^2 - 3x}{x^2 + x - 12}$$

Analyze the rational function with a Hole

$$\frac{2x^2 - 5x + 2}{x^2 - 4}$$

### Finding the Least Cost

Reynolds Metal Company manufactures aluminum cans in the shape of a cylinder with a capacity of 500 cubic centimeters ( $\text{cm}^3$ ), or  $\frac{1}{2}$  liter. The top and bottom of the can are made of a special aluminum alloy that costs  $0.05\phi$ /per square centimeter ( $\text{cm}^2$ ). The sides of the can are made of material that costs  $0.02\phi/\text{cm}^2$ .

- (a) Express the cost of material for the can as a function of the radius  $r$  of the can.
- (b) Use a graphing utility to graph the function  $C = C(r)$ .
- (c) What value of  $r$  will result in the least cost?
- (d) What is this least cost?