Precalculus
Lesson: 4.3 Complex Zeros; Fundamental Theorem of Algebra
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Not all quadratic equations have real solutions. If we look at the complex number system, every quadratic equation has at least one solution; note rational and irrational roots are in fact complex numbers. We just don't write them in the complex form. The fact that each polynomial function will have a complex brings about an important theorem.

Fundamental Theorem of Algebra.

Every complex polynomial function $f(x)$ of degree $n \geq 1$ has at least one complex zero.

Another important theorem states that if we have the solution $a + bi$ then we must also have the solution $a - bi$.

Conjugate Pairs Theorem.

Let $f(x)$ be a polynomial function whose coefficients are real numbers. If $r = a + bi$ is a zero of f, the complex conjugate $\bar{r} = a - bi$ is also a zero of f.

Using the Conjugate Pairs Theorem

A polynomial function f of degree 5 whose coefficients are real numbers has the zeros 1, $5i$, and $1 + i$. Find the remaining two zeros.

(a) Find a polynomial function f of degree 4 whose coefficients are real numbers and that has the zeros 1, 1, and $-4 + i$.

(b) Graph the function found in part (a) to verify your result.

Find the complex zeros of the polynomial function:

$$f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$$