Precalculus
Lesson 4.2: The Real Zeros of a Polynomial Function
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Dividing polynomials is a very similar process to the old long divisions we did in elementary school:

$$\frac{38}{7} = 5 + \frac{3}{7} \quad where \frac{3}{7} \; is\ the\ fractional\ remainder$$

Proper format for division of polynomials:

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

Dividend

or

Remainder

$$f(x) = q(x)g(x) + r(x)$$

Divisor

Quotient

Divide:	Divide:
$6x^2 - 26x + 12 \;\; by\ x - 4$	$P(x) = 8x^4 + 6x^2 - 3x + 1$ $D(x) = 2x^2 - x + 2$

Synthetic Division

This is a quick method for dividing polynomials **when the divisor is of the form $x - c$**

Divide: $2x^3 - 7x^2 + 5 \quad by \quad x - 3$

The Remainder Theorem

If the polynomial f(x) is divided by $x - c$, then the remainder is the value $f(c)$.

Find the remainder if $f(x) = x^3 - 4x^2 - 5$ use synthetic division and check using the remainder theorem.

1. divide by $x - 3$
2. divide by $x + 2$

Factor Theorem

$$Let\ f\ be\ a\ polynomial\ funciton.\ Then\ (x-c)\ is\ a\ factor\ of\ f(x)$$
$$if\ and\ only\ if\ f(c)=0$$

Basically if $f(c)=0,\ \ then\ \ x-c\ is\ a\ factor\ of\ f(x)\ \ and\ \ \ c\ is\ a\ zero\ of\ f(x)$

Use the factor theorem to determine whether the function

$$f(x)=2x^3-x^2+2x-3$$

has the factor

a) $x-1$
b) $x+2$

Finding the roots

Take a look at $P(x)=(x-2)(x-3)(x+4)$ multiplying the factors together we get:

$P(x)=x^3-x^2-14x+24$ Where did the constant 24 come from?

So, the constants of the factors multiplied out give us the constant of P(x). If the product of the zeros equals the constant then the zeros are all factors of the constant! We use this fact in the form of the **Rational Zeros Theorem**.

Rational Zeros Theorem

If the polynomial, P, has integer coefficients,

then every rational zero of P is of the form $\boldsymbol{\frac{p}{q}}$

p is a factor of the constant coefficient
q is a factor of the leading coefficient.

a. *So you need to find all the possible* $\pm p\ values\ and\ \pm q\ values\ to\ make\ all\ the\ \pm\frac{p}{q}\ ratios$ *One or more of the* $\pm\frac{p}{q}$ *ratios will be zeros of the polynomial.* *Determine zeros either by using the remainder theorem:* $P\left(\frac{p}{q}\right)=0$ *, or if a calculator is allowed, use the graphing feature to locate the rational roots.*
b. *Once you find a zero, use synthetic division to reduce the polynomial into factors.*
c. *Keep following this process until you reach a quadratic factor then factor the quadratic, or use the Quadratic Formula to calculate last two factors.*

With graphing calculators we don't have to use the remainder theorem to find rational factors.. Remember! If you are not allowed to use a graphing calculator, you will want to plug into the polynomial each possible zero to see which one yields a remainder of zero, hence, a factor.

Steps for finding the real zeros:

1. List all possible $\frac{p}{q}$ ratios in reduced form.
2. Graph to see which ratios are zeros of the polynomial. If no calculator is allowed, use the remainder theorem to make this determination.
3. The degree of the polynomial indicates the number of zeros of a function.
4. Use synthetic division to pare down the function to a quadratic factor and linear factors. **see below*
5. With the factors you can determine the zeros of the function.
6. If the quadratic factor is considered to be **irreducible or prime** if it cannot be factored over the real numbers.
7. If all real roots are expected then use the Quadratic formula on the irreducible quadratic to find the irrational roots (real) or the not so real imaginary roots. See 4.4.

* When the polynomial is divided by one of its binomial factors the resulting quotient is known as a **depressed polynomial.** Go figure! The depressed polynomial may be broken down in to other binomial factors if there are other rational roots.

1. List the possible rational zeros for the function below.
2. Then find the real zeros of the polynomial.
3. Factor completely.

$$f(x) = 2x^3 + 11x^2 - 7x - 6$$

Solve the equation:

$$3x^3 + 4x^2 - 7x + 2 = 0$$