Precalculus
Lesson 2.3: Properties of Functions
Lesson 2.4: Library of Functions; Piecewise-defined Functions
Mrs. Snow, Instructor

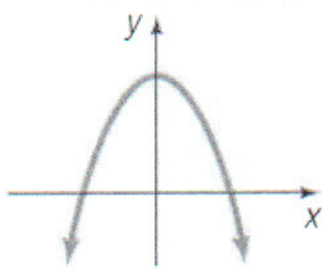
Even and Odd Functions

Think back to algebra, how do we determine the degree of a function?

- A function is **even** if and only if its graph is symmetric with respect to the y-axis.
- Whenever the point (x, y) is on the graph of f then the point $(-x, y)$ is also on the
- $f(-x) = f(x)$

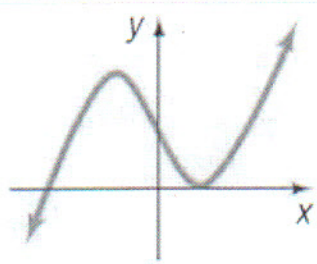
- A function is **odd** if and only if its graph is symmetric with respect to the origin.
- For every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.
- $f(-x) = -f(x)$

Identify if the function is even, odd or neither

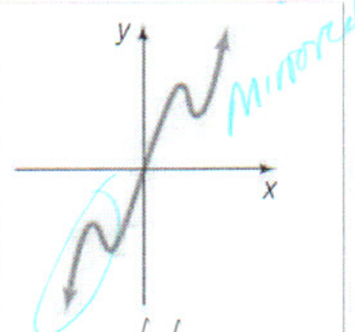


even

symmetry



neither



mirrored

odd

Even or Odd?

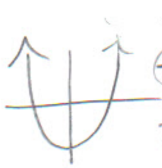
$f(x) = x^2 - 5$

↑ exp = 2

$f(-x) = f(x)$

$(-x)^2 - 5 = x^2 - 5$

$x^2 - 5 = x^2 - 5$ ✓



even

$g(x) = x^3 - 1$

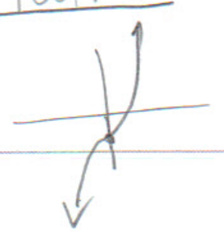
↑ exp = 3 ?

$f(-x) = -f(x)$

$(-x)^3 - 1 \stackrel{?}{=} -(x^3 - 1)$

$-x^3 - 1 \neq -(x^3) + 1$

Neither



$h(x) = 5x^3 - x$?

$f(-x) = -f(x)$

$5(-x)^3 - (-x) \stackrel{?}{=} -(5x^3 - x)$

$-5x^3 + x = -5x^3 + x$ ✓

odd

Increasing, Decreasing, or Constant?

Functions are often used to model changing quantities. Where the slope is positive we say it is increasing, and where the slope is negative we say it is decreasing. Horizontal? It is neither increasing nor decreasing; it is considered constant.

Determine where a function is increasing, decreasing or constant:

Where is the function increasing?

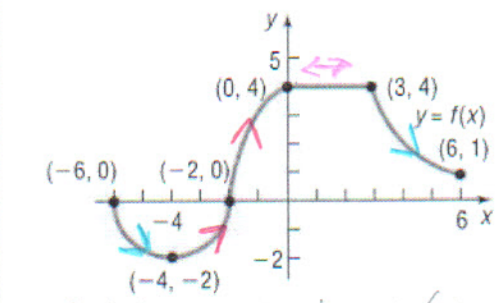
(4, 0)

Where is the function decreasing?

(-6, 4) ∪ (3, 6)

Where the function constant?

(0, 3)



what is happening at (4, -2)?
use an open interval to describe independent variable => $a<x<b$

Maximum and Minimum

<u>Absolute and Local</u>

DEFINITION Let f denote a function defined on some interval I. If there is a number u in I for which $f(x) \leq f(u)$ for all x in I, then $f(u)$ is the **absolute maximum of** f on I and we say **the absolute maximum of** f **occurs at** u.

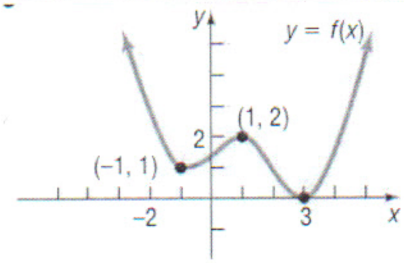
If there is a number v in I for which $f(x) \geq f(v)$ for all x in I, then $f(v)$ is the **absolute minimum of** f on I and we say **the absolute minimum of** f **occurs at** v.

Note: The interval I is closed, if it is open, then there can be no absolute minimum or maximum.

A function f has a **local maximum** at c if there is an open interval I containing c so that for all x in I, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum value of** f.

A function f has a **local minimum** at c if there is an open interval I containing c so that, for all x in I, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum value of** f.

Note: Local maximum and minimum occur in open intervals.

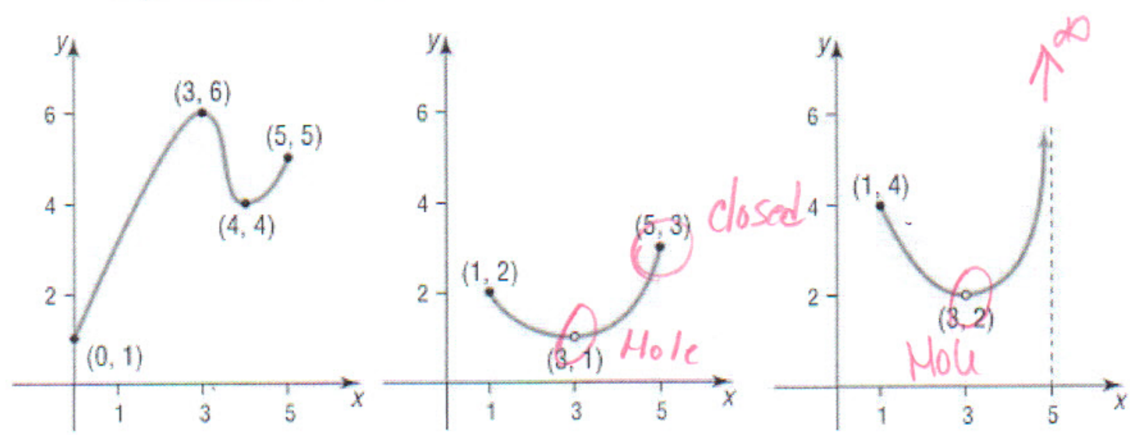


(a) At what value(s) of x, if any, does f have a local maximum? List the lo maximum values. Local max at 1 ; Local max $f(1)=2$

(b) At what value(s) of x, if any, does f have a local minimum? List the lo minimum values. Local min at -1; $f(-1)=1$ Local min at 3; $f(3)=0$

(c) Find the intervals on which f is increasing. Find the intervals on which f decreasing. Increasing $(-1,1) \cup (3,\infty)$ Decreasing $(-\infty,1) \cup (1,3)$

Find absolute maxima and absolute minima, if they exist



Absolute max at 3
$f(3)=6$

Absolute min at 4
$f(4)=4$

Abs. max at 5
$f(5)=3$

abs. min

no abs. max.

no abs. min.

Calculators often help in determining absolute maxima and minima and find where and find where a function is decreasing or increasing.

Use a graphing utility to graph $f(x) = 6x^3 - 12x + 5$ for $-2 < x < 2$. Approximate where f has a local maximum and where f has a local minimum.

Loc. max at $-.816$
$f(-.816) = 11.53$

Loc. min at $.816$
$f(.816) = -1.53$

Average Rate of Change

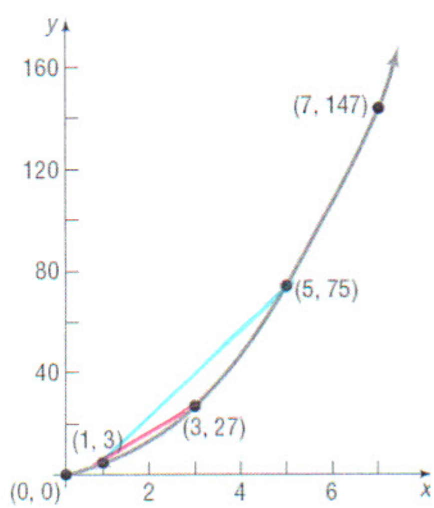
$$Average\ rate\ of\ change\ = \frac{\Delta y}{\Delta x} = \frac{\boldsymbol{f(b) - f(a)}}{\boldsymbol{b - a}} = \boldsymbol{m_{sec}}$$

When a function is graphed and the *average rate of change* is calculated, graphically a line drawn between $f(a) and\ f(b)$ is called a **secant line**. The *average rate of a secant line is designated as:* $\boldsymbol{m_{sec}}$

Find the average rate of change of $f(x) = 3x^2$:

a) From 1 to 3

b) From 1 to 5



ⓐ $\frac{\Delta y}{\Delta x} = \frac{27-3}{3-1} = \frac{24}{2} = 12$

ⓑ $\frac{\Delta y}{\Delta x} = \frac{75-3}{5-1} = \frac{72}{4} = 18$

Lesson 2.4

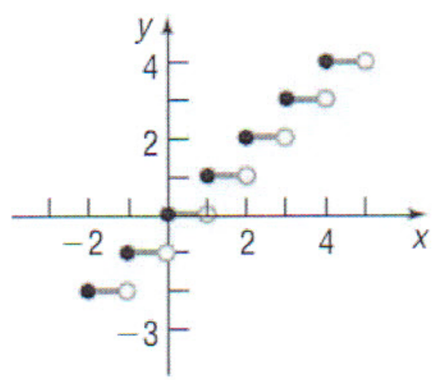
Step Function: A function whose graph is defied by the greatest integer function! WHAT!

Greatest Integer Function

$$f(x) = \text{int}(x)^* = \text{greatest integer less than or equal to } x$$

To graph this function, start by building a table of **x** values. Make sure to include negatives and fractions. Use the above definition to determine the output value of $f(x)$.

Table:



Is this a function? Vertical line test? yes-function

Is it continuous or discontinuous?

breaks in graph

continuous: a function in which its graph has no gaps of holes in it. It can be drawn without lifting your pencil from the paper.

discontinuous: a function in which its graph has holes or breaks in the curve. To draw this graph you would need to lift your pencil off the paper.

Piecewise Function: a function that is defined by different formulas on different parts of the domain.

graph:

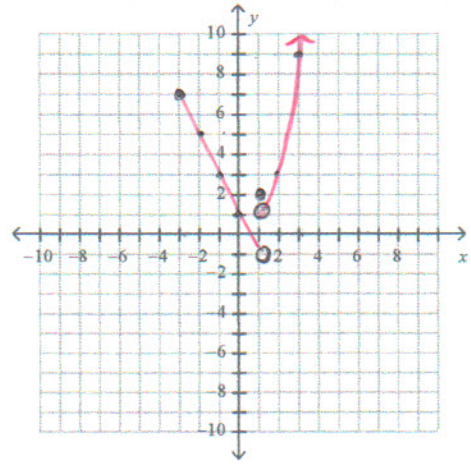
$$f(x) = \begin{cases} -2x+1 & if\ -3 \le x < 1 \\ 2 & if\ x = 1 \\ x^2 & if\ x > 1 \end{cases}$$

Find $f(-2) = 5$

$f(1) = 2$

$f(2) = 4$

* Which piece of the function?



What is the domain? Range?

D: $[-3, \infty)$ R: $(-1, \infty)$

Locate intercepts

$(0, 1)$ $(\frac{1}{2}, 0)$

Is f continuous over its domain?

discontinuous

$0 = -2x + 1$

$-1 = -2x$

$\frac{1}{2} = x$