Precalculus
Lesson 2.1 What is a Function
Lesson 2.2 Graphs of Functions
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Lesson 2.1

Before we look at functions we first need to recognize a broader group called **relation**. A **relation** is a set of ordered pairs. So the following are relations because they consist of a set of ordered pairs.

$A)\ \{(0,5),(2,5),(4,-3),(7,12),(9,-9),(10,-3)\}$
$B)\ \{(0,5),(2,5),(4,8),(4,0),(0,-4),(8,12)\}$

A **function** is a relation for which each value from the set the first components of the ordered pairs is associated with exactly one value from the set of second components of the ordered pair.

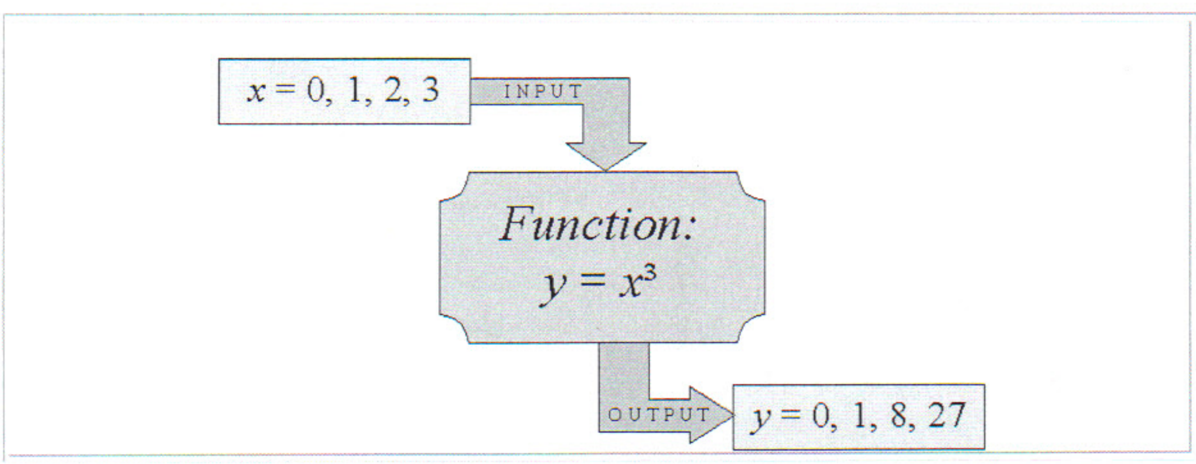
Okay, that's quite a mouth full. What does it all mean with our examples of relations above? If we separate the 1st component which we traditionally identify as the x-element and the 2nd component A.K.A. y-element, into sets we get:

$A)\ 1st\ components\colon \{0,2,4,7,9,10\}\ and\ 2nd\ components\colon \{-9,-3,5,12,\}$
$B)\ 1st\ components\colon \{0,2,4,4,0,8\}\ and\ 2nd\ components\colon \{-4,0,5,8,12\}$

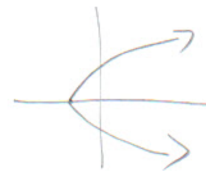
What we should see is that in example B the number 4 is repeated. In example A there are no repeaters in the 1st components. Therefore, the relation A is also a function, while the relation B is merely a relation and NOT a function. Note there are no repeating 1st components or what we say, no repeating independent x values.

"Working Definition" of Function"

A **function** is a relation for which each value from the set the first components of the ordered pairs is associated with exactly one value from the set of second components of the ordered pair. When we think of function equations, for every input x there exactly one output value of $x's$.



Determine whether the equation is a function.

$y = \frac{1}{2}x - 3$	$x = y^2 - 1$
$y = mx + b$ form function	$y^2 = x + 1$ $y = \pm\sqrt{x+1}$  Not function

For the given function evaluate: $f(x) = 2x^2 - 3x$ for:

(a) $f(3)$ (b) $f(x) + f(3)$ (c) $3f(x)$ (d) $f(-x)$

(e) $-f(x)$ (f) $f(3x)$ (g) $f(x+3)$ (h) $\dfrac{f(x+h) - f(x)}{h}$ $h \neq 0$

(a)

$f(3) = 2(3^2) - 3(3)$

$= 18 - 9 = \underline{\underline{9}} = f(3)$

(b)

$f(x) + f(3) = 2x^3 - 3x + 9$

(c) $3f(x) = 3(2x^2 - 3x)$

$= 6x^2 - 9x$

(d) $f(-x) = 2(-x)^2 - 3(-x)$

$= 2x^2 + 9$

(e) $-f(x) = -(2x^2 - 3x)$

$= -2x^2 + 9x$

(f) $f(3x) = 2(3x)^2 - 3(3x)$

$= 2(9x^2) - 9x$

$= 18x^2 - 9x$

(g) $f(x+3) = 2(x+3)^2 - 3(x+3)$

$= 2(x^2 + 6x + 9) - 3x - 9$

$= 2x^2 + 1\cancel{2}x + 18 - \cancel{3}x - 9$

$= 2x^2 + 9x + 9$

(h) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$

$= \dfrac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h - \cancel{2x^2} + \cancel{3x}}{h}$

$= \dfrac{2h^2 + 4xh - 3h}{h}$

$= \dfrac{\cancel{h}(2h + 4x - 3)}{\cancel{h}} = \underline{\underline{4x + 2h - 3}}$

Domain of a Function

Three points to remember!!

1. Denominator cannot equal zero
2. Anything under a square root has to be greater than or equal to zero
3. If no domain is specified, then the domain will be taken to be the largest set of real numbers for which the equation defines a real number.

Find the domain: *Remember interval notation only!!!*

$f(x) = \dfrac{x+4}{x^2-2x-3}$ $\neq 0$ $(x-3)(x+1) \neq 0$ $x-3 \neq 0 \quad x+1 \neq 0$ $x \neq 3 \quad x \neq -1$ D: $(-\infty,-1) \cup (-1,3) \cup (3,\infty)$	$g(x) = x^2 - 9$ All Real $= (-\infty,\infty)$ (Parabola graph)	$h(x) = \sqrt{3-2x}$ ≥ 0 $3-2x \geq 0$ $-2x \geq -3$ $x \leq \frac{3}{2}$ $(-\infty, \frac{3}{2}]$

If we have two functions, we can use different techniques to combine them into one function

If f and g are functions:

The **sum** $f + g$ is the function defined by

$$(f+g)(x) = f(x) + g(x)$$

Domain: $f \cap g$

The **difference** $f - g$ is the function defined by

$$(f-g)(x) = f(x) - g(x)$$

Domain: $f \cap g$

The **product** $f \cdot g$ is the function defined by

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Domain: $f \cap g$

The **quotient** $\dfrac{f}{g}$ is the function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0$$

Domain: $\{x \mid g(x) \neq 0\}$, $\cap$ domain of f $\cap$ domain of g

Combinations of Functions and Their Domains:

Let $f(x) = 2x^2 + 3 \quad and\ g(x) = 4x^3 + 1$

1. Find the functions $(f+g)(x),\ (f-g)(x),\ (f \cdot g)(x), and\ \left(\frac{f}{g}\right)(x)$ and determine their domains.

$(f+g)(x) = 2x^2+3+4x^3+1$

$= 4x^3+2x^2+4$

D: $(-\infty, \infty)$

$(f-g)(x) = 2x^2+3-(4x^3+1)$

$= -4x^3+2x^2+2$

D: $(-\infty, \infty)$

$(f \cdot g)(x) = (2x^2+3)(4x^3+1)$

$= 8x^6+12x^3+2x^2+3$

D: $(-\infty, \infty)$

$\left(\frac{f}{g}\right)(x) = \dfrac{2x^2+3}{4x^3+1}$

$4x^3+1 \neq 0$

$4x^3 = -1$

$x^3 = \frac{-1}{4}$

$x \neq \sqrt[3]{\frac{-1}{4}}$

$x \neq -.62996$

D:

$(-\infty, -.63) \cup (-.63, \infty)$

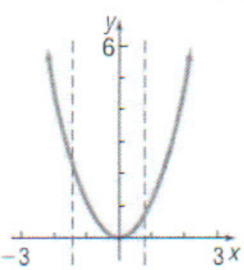
Lesson 2.2

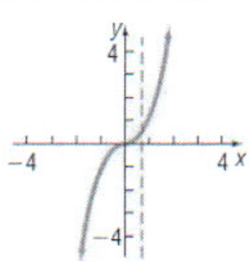
Sometimes a visual representation, a graph, of a relationship is easier to understand.

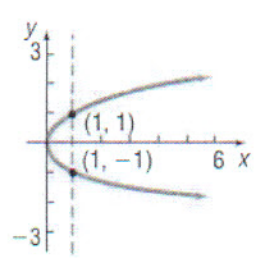
The Vertical Line Test is a technique to verify if a graph represents a function.

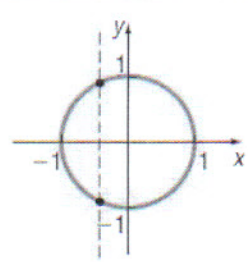
Vertical Line Test: The graph of a function cannot contain two points with the same x-coordinate and different y-coordinates.

Identify the graphs that represent a function and the domains for all:

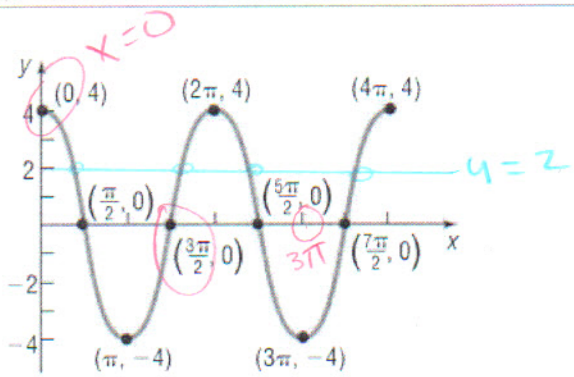








Obtaining Information from the Graph of a Function



(a) What are $f(0)$, $f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$? $f(0)=4$, $f\left(\frac{3\pi}{2}\right)=0$, $f(3\pi)=-4$

(b) What is the domain of f? All Real $(-\infty, \infty)$

(c) What is the range of f? $[-4, 4]$

(d) List the intercepts. (Recall that these are the points, if any, where the graph crosses or touches the coordinate axes.) $(0,4)$, $(\frac{\pi}{2},0)$, $(\frac{3\pi}{2},0)$, $(\frac{5\pi}{2},0)$, $(\frac{7\pi}{2},0)$

(e) How many times does the line $y = 2$ intersect the graph? 4 times

(f) For what values of x does $f(x) = -4$? $(\pi,-4)$, $(3\pi,-4)$

(g) For what values of x is $f(x) > 0$?

$[0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, \frac{5\pi}{2}) \cup (\frac{7\pi}{2}, 4]$

Obtaining Information about the Graph of a Function

Consider the function: $f(x) = \dfrac{x+1}{x+2}$

(a) Find the domain of f. $x+2 \neq 0$ so $x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$

(b) Is the point $\left(1, \frac{1}{2}\right)$ on the graph of f? NO

(c) If $x = 2$, what is $f(x)$? What point is on the graph of f?

(b) $\frac{1}{2} \stackrel{?}{=} \frac{1+1}{1+2}$

$\frac{1}{2} \neq \frac{2}{3}$ NO

(c) $f(x) = \frac{2+1}{2+2}$

$f(x) = \frac{3}{4}$

Average Cost Function

The average cost $\overline{C}$ of manufacturing x computers per day is given by the function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

Determine the average cost of manufacturing:

(a) 30 computers in a day

(b) 40 computers in a day

(c) 50 computers in a day

(d) Graph the function $\overline{C} = \overline{C}(x)$, $0 < x \leq 80$.

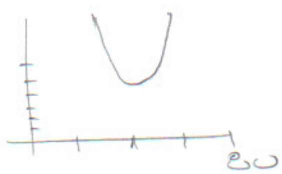
(e) Create a TABLE with TblStart = 1 and ΔTbl = 1. Which value of x minimizes the average cost?

a at $x = 3$ $C(\bar{x}) =$ \$1351.64

b 40/day $C(\bar{x}) =$ \$1232.97

50/day $C(\bar{x}) =$ \$1293.07

c



d

x	\$	y
40	1233	
41	\$1231.70	
42	\$1232.20	

← minimum cost at 41 computers.