Precalculus Name______________________________ per____
The Chain Rule

This packet will show you how to find the derivatives of composite functions. A composite function can be represented as $f(g(x))$ with $f$ being the outside function and $g$ being the inside.

Examples of composite functions:

$y=(4x^2+1)^7$ the outside function is $f(x)=x^7$

the inside function is $g(x)=4x^2+1$

$y=f(g(x))=(4x^2+1)^7$

$y=e^{3x}$ the outside function is $f(x)=e^x$

the inside function is $g(x)=3x$

$y=f(g(x))=e^{3x}$

## The Chain Rule

**The Chain Rule**

$$\frac{d}{dx}f(g(x))=f'(g(x))\cdot g'(x)$$

The derivative of a composite function is the product of the derivatives of the outside and inside functions. The derivative of the outside function must be evaluated at the inside function.

**Example 1**. Differentiate $y=(4x^2+1)^7$

*Solution*: the outside function is $x^7$ and $\frac{d}{dx}x^7=7x^6$

Remember to evaluate at $4x^2+1$

the inside function is $4x^2+1$ and $\frac{d}{dx}(4x^2+1)=8x$

$y'=7(4x^2+1)^6\cdot 8x$

$=56x(4x^2+1)^6$

**Example 2.** Differentiate $y=e^{3x}$

*Solution*: the outside function is $e^x$ and $\frac{d}{dx}e^x=e^x$

the inside function is $3x$ and $\frac{d}{dx}3x=3$

Remember to evaluate at $3x$

$y'=e^{3x}\cdot 3$

$=3e^{3x}$

**Example 3.** Differentiate $f(x)=\sqrt{3x^2+5x-2}$

*Solution*: the outside function is $\sqrt{x}$ and $\frac{d}{dx}\sqrt{x}=\frac{1}{2\sqrt{x}}$

the inside function is $3x^2+5x-2$ and $\frac{d}{dx}(3x^2+5x-2)=6x+5$

$$f'(x)=\frac{1}{2\sqrt{3x^2+5x-2}}\cdot(6x+5)=\frac{6x+5}{2\sqrt{3x^2+5x-2}}$$

## Using the Product and Chain Rules to Differentiate

**Example 4.** Differentiate $k(x)=\dfrac{x}{\left(x^2+1\right)^2}$

*Solution*: write the original function as a product $k(x)=\dfrac{x}{\left(x^2+1\right)^2}=x\cdot\left(x^2+1\right)^{-2}$

now use the product rule to differentiate $k'(x)=\dfrac{d}{dx}\left(x\cdot\left(x^2+1\right)^{-2}\right)=1\cdot\left(x^2+1\right)^{-2}+x\cdot\dfrac{d}{dx}\left(x^2+1\right)^{-2}$

now use the chain rule to differentiate $\dfrac{d}{dx}\left(x^2+1\right)^{-2}$

$$=\left(x^2+1\right)^{-2}+x\cdot\left(-2\left(x^2+1\right)^{-3}\cdot 2x\right)$$
$$=\frac{1}{\left(x^2+1\right)^2}+\frac{-4x^2}{\left(x^2+1\right)^3}$$

**Example 5.** Differentiate $y=te^{-t^2}$

*Solution*: $y'=1\cdot e^{-t^2}+t\cdot\dfrac{d}{dt}e^{-t^2}$

$$=e^{-t^2}+t\cdot\left(e^{-t^2}\cdot -2t\right)$$
$$=e^{-t^2}+-2t^2e^{-t^2}$$
$$=\left(1-2t^2\right)e^{-t^2}$$

**Example 6.** Differentiate $f(x)=\left(2x^3-5x^2\right)\left(e^{3x}+1\right)$

*Solution*: $f'(x)=\left(6x^2-10x\right)\left(e^{3x}+1\right)+\left(2x^3-5x^2\right)\dfrac{d}{dx}\left(e^{3x}+1\right)$

$$=\left(6x^2-10x\right)\left(e^{3x}+1\right)+\left(2x^3-5x^2\right)\left(\left(e^{3x}\right)\cdot 3\right)$$
$$=\left(6x^2-10x\right)\left(e^{3x}+1\right)+\left(6x^3-15x^2\right)e^{3x}$$
$$=6x^2e^{3x}+6x^2-10xe^{3x}-10x+6x^3e^{3x}-15x^2e^{3x}$$
$$=e^{3x}(6x^3+6x^2-15x^2-10x)+6x^2-10x$$
$$=e^{3x}(6x^3-9x^2-10x)+6x^2-10x$$

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Chain Rule

Section 1. For Problems 1-12, find the derivative **using the chain rule**. (5 points each)

1. $f(x)=(x+1)^{90}$ 1. ________________________

2. $f(x)=\sqrt{1-x^2}$ 2. ________________________

3. $y=(t^2+1)^{100}$ 3. ________________________

4. $y=(\sqrt{x}+1)^{100}$ 4. ________________________

5. $y=e^{2t}$ 5. ________________________

6. $w=\dfrac{1}{x^2+x^4}$ 6. ________________________

7. $f(t)=2^{5t-3}$ 7. ________________________

8. $y=e^{3w/2}$ 8. ________________________

9. $w=e^{\sqrt{s}}$ 9. ________________________

10. $y=\sqrt{x^3+1}$ 10. ________________________

11. $f(x)=\left(3x^5+6x^2-7\right)^4$ 11. ____________________

12. $f(x)=\frac{1}{x^3+5x}$ 12. ____________________

Section 2. For Problems 13-16, find the derivative **using the product and chain rules**. (10 points each)

13. $f(x)=\left(5x^2+3\right)e^{x^2}$ 13. ____________________

14. $y=te^{5-2t}$ 14. ____________________

15. $f(z)=\frac{z}{\left(e^z+1\right)^2}$ 15. ____________________

16. $f(x)=\left(x^2+3x\right)\left(1-e^{-2x}\right)$ 16. ____________________