Precalculus
Lesson 000: Stuff You Should Already Know: Algebra II Review
Mrs. Snow, Instructor

Polynomials

A **polynomial** in one variable is an algebraic expression of the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants,* called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is a variable. If $a_n \neq 0$, it is the **leading coefficient,** and n is the **degree** of the polynomial.

$$-8x^3 + 4x^2 + 6x + 2$$

a) What is the degree of the polynomial?

b) What is the leading coefficient?

Special products

Difference of Two Squares

$$(x - a)(x + a) = x^2 - a^2 \qquad \textbf{(2)}$$

Squares of Binomials, or Perfect Squares

$$(x + a)^2 = x^2 + 2ax + a^2 \qquad \textbf{(3a)}$$

$$(x - a)^2 = x^2 - 2ax + a^2 \qquad \textbf{(3b)}$$

Cubes of Binomials, or Perfect Cubes

$$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3 \qquad \textbf{(4a)}$$

$$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3 \qquad \textbf{(4b)}$$

Difference of Two Cubes

$$(x - a)(x^2 + ax + a^2) = x^3 - a^3 \qquad \textbf{(5)}$$

Sum of Two Cubes

$$(x + a)(x^2 - ax + a^2) = x^3 + a^3 \qquad \textbf{(6)}$$

Multiply the factors:

$(x-3)(x+3)$	$(x+2)^2$
$(2x+1)(3x+4)$	$(x-2)(x^2+2x+4)$

Factor:

x^4-16	x^3-1

$9x^2 - 6x + 1$	$x^2 + 4x - 12$
$3x^2 + 10x - 8$	$x^3 - 4x^2 + 2x - 8$

Simplifying Rational Expressions:

$\dfrac{x^2 + 4x + 4}{x^2 + 3x + 2}$	$\dfrac{x^3 - 8}{x^3 - 2x^2}$

$$\frac{8-2x}{x^2-x-12}$$	

Multiplying and Dividing Rational Expressions

$$\frac{x^2-2x+1}{x^3+x}\cdot\frac{4x^2+4}{x^2+x-2}$$	$$\frac{\frac{x+3}{x^2-4}}{\frac{x^2-x-12}{x^3-8}}$$

Adding and Subtracting Rational Expressions

$$\frac{x^2}{x^2-4}-\frac{1}{x}$$	$$\frac{x}{x^2+3x+2}+\frac{2x-3}{x^2-1}$$

Simplify:

$$\frac{\frac{1}{2}+\frac{3}{x}}{\frac{x+3}{4}}$$

Quadratic Formula

Quadratic Formula

Consider the quadratic equation

$$ax^2 + bx + c = 0 \quad a \neq 0$$

If $b^2 - 4ac < 0$, this equation has no real solution.
If $b^2 - 4ac \geq 0$, the real solution(s) of this equation is (are) given by the **quadratic formula.**

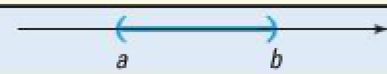
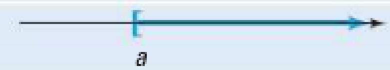
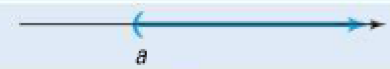
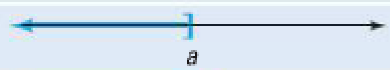
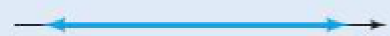
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the solutions, if any, of the equation:

$$3x^2 - 5x + 1 = 0$$

Interval Notation

Let a and b represent two real numbers with $a < b$.
A **closed interval,** denoted by **[a, b]**, consists of all real numbers x for which $a \le x \le b$.
An **open interval,** denoted by **(a, b),** consists of all real numbers x for which $a < x < b$.
The **half-open,** or **half-closed, intervals** are **(a, b],** consisting of all real numbers x for which $a < x \le b$, and **[a, b),** consisting of all real numbers x for which $a \le x < b$.

Interval	**Inequality**	**Graph**
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \le x \le b$	
The half-open interval $[a, b)$	$a \le x < b$	
The half-open interval $(a, b]$	$a < x \le b$	
The interval $[a, \infty)$	$x \ge a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \le a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

Write each inequality using interval notation $1 \le x \le 3$	$-4 < x < 0$
$x > 5$	$x \le 1$