FACTORING – TAKING POLYNOMIALS APART **DUE EXAM DAY**

NAME____ **CLASS PD**_____

The whole reason to factor a polynomial is that it is an algebraic method to find the solutions to a quadratic equation.

What are Prime numbers?_________________________

List the prime number starting with 1 ____

The "L" method of factoring.

Number 300300

1	*	300300
2	*	150150
2	*	75075
3	*	25025
5	*	5005
5	*	1001
7	*	143
11	*	13 (P)

Number 56

-1	*	56
1	*	56
2	*	28
2	*	14
2	*	7

The numbers in the "L" are the prime factors .

Factoring Variables: break apart each variable and when a coefficient is present, break it apart using the "L" method.

X^3	$x^3y^2z^3$	$12x^2yz^3$	
X	x y z	1*12	x y z
X	x y z	2*6	x z
X	x z	2 *3	z

So what do you have in common? Circle the common stuff.

You do it:

468 -9282 945 -12530 320

$36x^3y^6z^3$

$5022x^6yz^2$

When you factor a polynomial, you use the L method to factor each term, then gather up all the common factors for lunch and leave the leftovers. (we will presume the 1 factor)

$(12x^2yz^3) + (24x^3yz^4) - (8xyz^3)$

Gather up the common factors for lunch (circle them)
lunch: $2 \cdot 2 \cdot x \cdot y \cdot zzz \ \ or\ 4xyz^3$
leftovers: $3x + 2 \cdot 3 \cdot xx \cdot z - 2 \ \ or\ \ 3x + 6x^2z - 2$
we now write as: $4xyz^3(3x + 6x^2z - 2)$

Factoring is really Un-distributing – the inverse of distribution.

Greatest Common Factor: The "lunches"are the greatest common factors of the terms. In other words: Greatest common factor is the biggest number or variable power that "goes into" each term evenly.

Find the GCF of each pair of monomials.

15. $15x^4$ and $35x^2$ ________________

16. $12p^2$ and $30q^5$ ________________

17. $-6t^3$ and $9t$ ________________

18. $27y^3z$ and $45x^2y$ ________________

19. $12ab$ and 12 ________________

20. $-8d^3$ and $14d^4$ ________________

21. $-m^8n^4$ and $3m^6n$ ________________

22. $10gh^2$ and $5h$ ________________

Before you start to factor 3 items to check for:

1. ***ALWAYS CHECK TO SEE IF YOU CAN FACTOR OUT A COMMON VALUE, IT WILL MAKE THE PROBLEM EASIER***
2. ***ALWAYS MULTIPLY THROUGH BY THE DENOMINATOR IF A FRACTION IS PRESENT***
3. ***ALWAYS MULTIPLY THROUGH BY A NEGATIVE 1 IF THE LEADING COEFFICIENT IS NEGATIVE***

Factor each polynomial. Check your answer.

1. $8x^4 - 12x^2$

2. $-12ab^3 + 20b$

3. $16m^2 - 2n^3 + 30m$

______ ______ ______

4. $27j^4 - 72j^3 + 9j$

5. $-5x^5 + 35x^4 - 30x^3$

6. $16x^6y + 16x^2y^4 + 32x^3y^2$

______ ______ ______

7. The expression used for finding the surface area of a cylinder is $2\pi r^2 + 2\pi rh$. Factor this expression. ______

8. The area of a hallway rug is $\frac{3}{2}x^2 + \frac{1}{2}x$ ft^2. Factor this polynomial to find expressions for the dimensions of the rug. ______

Factor each expression.

9. $10(k - 2) + 7k(k - 2)$

10. $9m^2(m + 7) + 5(m + 7)$

FACTORING QUADRATICS

There are several ways to factor quadratics and ultimately a quadratic equation. In this class we will only concentrate on 2 methods:

1. "Splitting down the Middle"
2. the Quadratic Formula

Splitting the Middle is limited to factors that have neat integers for coefficients and constants. It will not work for quadratic equations that have no real solutions.

- First the parts of a quadratic function:
- $y = ax^2 + bx + c$
- $a = \ leading\ coefficient,\ b = linear\ coefficient,\ and\ c = constant$

First a simple example:

$1x^2 + 2x - 35 = 0$	In case you did not remember, the leading coefficient is 1 1. Identify coefficients and constant
$a = 1, b = 2, c = -35$	2. Multiply $a \cdot c$ 3. Go to **y=** on your calculator, type in $y_1 = \frac{ac}{x}$
$ac = (1)(-35) = -35$ $(35 \cdot -1), (\mathbf{-5 \cdot 7})$	4. Press **2nd GRAPH;** look at the table generated. Find a factor pair that add up to b
$1x^2 - 5x \mid + 7x - 35 = 0$	5. Rewrite the linear term **b**, as 2 terns using the factor pair summing to **ac.**
$x(\underline{x-5}) + 7(\underline{x-5}) = 0$	6. Split the quadratic down the middle between the linear terms. 7. Factor the left side **and** factor the right side.
$(x-5)(x+7) = 0$ $AB = 0\ \rightarrow A = 0$ $or\ B = 0$	8. Factor all. The binomial from each factor must be the same or you have made a mistake. ` 9. You have successfully factored the quadratic.
$(x-5) = 0 \therefore x = 5$ $x + 7 = 0 \therefore x = -7$ Check by graphing.	10. The zero product property states that if a product equals zero then one of the factors is equal to zero. Use this property to solve for x.

REALLY?!?! This is a lot of work to do. When the leading coefficient =1, you can if you know what you are doing basically go straight from step 4 to step 7. ONLY DO THIS SHORT CUT IF YOU UNDERSTAND THE SPLIT DOWN THE MIDDLE WORKS FOR ALL QUADRATICS. .

$1x^2 + 2x - 35 = 0$ $a = 1, b = 2, c = -35$	1. Identify coefficients and constant 2. Go to **y=** on your calculator, type in $y_1 = \frac{ac}{x}$
$ac = (1)(-35) = -35$ $(35 \cdot -1), (\mathbf{-5 \cdot 7})$	4. Press **2nd GRAPH;** look at the table generated. Find a factor pair that adds up to b
$(\boldsymbol{x} \qquad)(\boldsymbol{x} \qquad) \ = \mathbf{0}$ $(x-5)(x+7) = 0\ and\ x = 5 or - 7\ as\ above$ Always check your work!!!!	5. Set up the template as shown 6. Put the numbers in that you found to work.

Splitting down the middle works best when the leading coefficient is a value other than 1.

$$6x^2 - 2x - 4 = 0$$

$$2(3x^2 - 1x - 2) = 0$$

$$(3x^2 - 1x - 2) = 0$$

$$a = 3, b = -1, c = -2$$

$$ac = -6$$

$$(-6 \cdot 1), (\mathbf{-3 \cdot 2})$$

$$3x^2 - 3x \;\Big|\; + 2x - 2 = 0$$

$$3x(x - 1) + 2(x - 1) = 0$$

$$(x - 1)(3x + 2) = 0$$

$$x - 1 = 0 \therefore x = 1$$

$$3x + 2 = 0 \therefore x = -\frac{2}{3}$$

Check by graphing


First are there any common factors? If yes, factor out the common factor. Recognize you can drop the common factor as long as it is just a constant (number)

1. Identify coefficients and constant
2. Multiply $a \cdot c$
3. Go to **y₁=** on your calculator, type in $y_1 = \frac{ac}{x}$
4. Press **2nd GRAPH;** look at the table generated. Find a factor pair that add up to b
5. Rewrite the linear term **b**, as 2 terms using the factor pair summing to **ac.**
6. Split the quadratic down the middle between the linear terms.
7. Factor the left side and factor the right side.
8. The binomial from each factor must be the same or you have made a mistake. Now factor out the binomial.
9. you have successfully factored the quadratic

In summary use the following "ten step formula" for solving quadratic equations:

You try:

$$2x^2 + 13x + 15 = 0$$

Step 1: ID "abc" $a =$ $b =$ $c =$

Step 2: $ac =$

Step 3: Calculator **Y₁=**$\frac{ac}{x}$

Step 4: Pair that adds up to b (,)

Step 5: rewrite linear term and split $2x^2 + 13x + 15 = 0$

Step 6: factor the left side

Step 7: and factor the right side:

Step 8: Factor again:

Step 9: Use the zero product property to solve:

Step 10: Solution is: $\{ \quad , \quad \}$

HOMEWORK: Factor each quadratic. SHOW ALL WORK!!!

1. $x^2 + 7x + 10$

2. $x^2 + 9x + 8$

3. $x^2 + 13x + 36$

4. $2x^2 + 13x + 15$

5. $3x^2 + 10x + 8$

6. $4x^2 + 24x + 27$

7. $-12x^2 - 35x - 18$

8. $-20x^2 + 29x - 6$

9. $-2x^2 + 5x + 42$

10.

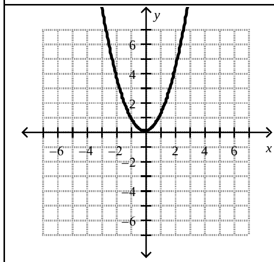
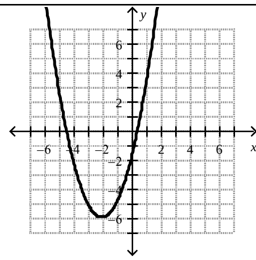
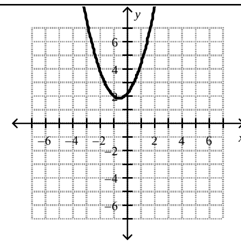
The area of a rectangle is $20x^2 - 27x - 8$.
The length is $4x + 1$. What is the width?

Right now we want to focus on factoring. Finding the solutions will come later. To find the "solutions" to a quadratic equation, you just set your factors = to zero and solve. Those are your solutions, roots, zeros, answers. Example if you got the two factors$(2x - 3)\ (4x + 5)$, to find the solutions:

$$2x - 3 = 0;\ x = \frac{3}{2}\ and\ \ 4x + 5 = 0;\quad x = -\frac{5}{4}$$

Written as the solution SET or $\left\{\frac{3}{2}, -\frac{5}{4}\right\}$

How many real solutions will a quadratic equation have?

1 solution	2 solutions	No *real* solutions
		

Looking at the graphs we see that there are 3 possible answers to our question. There is a way to determine the number of solutions to a quadratic. Determining the number of real solutions before attempting to solve a quadratic may save you time. This will be especially helpful if we have no solutions. We will know there are no real solutions that before we start to solve and use an appropriate method for solving.

If we calculate the **discriminant** we will know the number of solutions.

- Discriminant is positive – 2 solutions
- Discriminant is zero – 1 solution
- Discriminant is negative – No solutions

Cool, but what is the discriminant?

$$\text{Given } y = ax^2 + bx + c$$

$$\boldsymbol{Discriminant} = (\boldsymbol{b^2 - 4ac})$$

We can find the discriminant for our first example. We have 2 solutions, so the discriminant should be positive:

$1x^2 + 2x - 24 = 0$ $\qquad a = 1, b = 2, c = -35$

$$(b^2 - 4ac) = 2^2 - 4(1)(-24)$$
$$= 4 + 96 = \mathbf{100}$$

The discriminant is positive so yes, there are to be 2 solutions.

Try: Watch out for negative signs! 

$3x^2 - 1x - 2 = 0$ $a = 3, b = -1, c = -2$	$2x^2 + 3x + 5 = 0$	for $4x^2 + 12x + 9 = 0$

With a negative discriminant, we have no real solutions. No real solution means that the quadratic will not be factorable. Therefore **before attempting to factor always check the discriminant.**

QUADRATIC FORMULA

The Quadratic Formula is the Queen Bee of all methods for finding solutions to a quadratic equation.

Any quadratic may be solved using the quadratic formula.

For: $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What is the expression located under the radical? Yes, the discriminant. The reason why a negative discriminant will have no real solution is because a negative value under a radical takes us down the dark road of imaginary numbers. Imaginary numbers and negative discriminants are saved for Algebra 2

To solve a quadratic equation with the quadratic formula, you will follow the same process of identifying your a*, b, and c* values. Plug them into the equation and solve for x.

Solve: $3x^2 + 14x - 5 = 0$

$$a = 3, b = 14, c = -5$$

Discriminant:

$$14^2 - 4(3)(-5) = 196 - (-60) = 196 + 60 = 256$$

How many answers? ________________ So yes, there is a real solution.

Take the square root of 256 = 16

Now fill in the remaining part of the formula and solve: $x = \frac{-14 \pm 16}{2(3)}$

This gives us 2 equations to solve. Break them apart and solve separately.

$$x = \frac{-14 + 16}{6} \quad and\ x = \frac{-14 - 16}{6}$$

$$x = \frac{2}{6} = \frac{1}{3} \quad and\ x = -\frac{30}{6} = -5$$

Check: Solve by factoring: $3x^2 + 14x - 5 = 0 \quad a = 3,\ b = 14 \quad c = -5$

$$ac = -15$$

$$Y_1 = -\frac{15}{x}$$

$$(-1, 15)\ add\ up\ to + 14$$

$$3x^2 - 1x + 15x - 5 = 0$$

rewrite linear term and split

$$(3x^2 - 1x) + (15x - 5) = 0$$

factor left and right:

$$x(3x - 1) + 5(3x - 1) = 0$$

$$(3x - 1)(x + 5) = 0$$

zero product property: $3x - 1 = 0 \quad \therefore \quad x = \frac{1}{3}$

$$x + 5 = 0 \quad \therefore \quad x = -5$$

HOMEWORK:

Find the number of real solutions of each equation using the discriminant.

1. $+25 = 0$	2. $x^2 - 11x + 28 = 0$	3. $x^2 + 8x + 16 = 0$

Now finish solving (if possible) using the quadratic formula and write the solutions in set form and in factor form.

Solve using the quadratic formula. Show both factors and solutions using set notation.

1. $x^2 + x = 12$	2. $4x^2 - 17x - 15 = 0$	3. $2x^2 - 5x = 3$
4. $6x^2 + x - 1 = 0$	5. $x^2 + 8x - 20 = 0$	In the past, professional baseball was played at the Astrodome in Houston, Texas. The Astrodome has a maximum height of 63.4 m. The height of a baseball *t* seconds after it is hit straight up in the air with a velocity of 45 ft/s is given by $h = -9.8t^2 + 45t + 1$. Will a baseball hit straight up with this velocity hit the roof of the Astrodome? Use the discriminant to explain your answer.

PUTTING IT ALL BACK TOGETHER: MULTIPLYING BINOMIALS

Method 1. Old Fashioned Multiplication:

Multiply the binomials:	Do this like an old fashioned multiplication problem:
$(2x+3)(5x-9)$ $2x + 3$ $\times \quad \underline{5x - 9}$ $-18x - 27$ $\underline{10x^2 + 15x \quad}$ $10x^2 - 3x - 27$	• Multiply -9 $times\ 3 = 27$ • Multiply -9 $times\ 2x = 18x$ • Multiply $5x$ $times\ 3 = 15x$ • Multiply $5x$ $times\ 2x = 10x^2$. • Combine like terms by adding -$18x\ and\ +15x = -3x$

Method 2. FOIL:

$(2x+3)(5x-9) =$	**F O I L**
$2x \times 5x = 10x^2$ $2x \times (-9) = -18x$ $3 \times 5x = 15x$ $3 \times (-9) = -27$ $= 10x^2 - 3x - 27$	**First Outside Inside Last** • Multiply the first terms • Multiply the outside terms • Multiply the inside terms • Multiply the last terms • Combine like terms

- **<u>TAKS TIP: Put the problem in the calculator at Y1</u>**
- **<u>Put an answer at Y2</u>**
- **<u>Press 2nd graph of table. If ALL the y values match…it is correct.</u>**

Method 3. Distribution of the first polynomial. This works for quadratics, cubics, anything!!!

$$(2x+3)(5x-9) = 2x(5x-9) + 3(5x-9) = 10x^2 - 18x + 15x - 27$$
$$= 10x^2 - 3x - 27$$

This is a good method to know as foil only works for binomials and the multiplication method gets sloppy for trinomials and above, but this always works.

Example $(2x+5)(10x^2 - 3x - 27) = 2x(10x^2 - 3x - 27) + 5(10x^2 - 3x - 27)$

Multiply and simplify……

HOMEWORK – DUE EXAM DAY:

Multiply:

1. $(x+3)(x+4)$

2 $(x-6)(x-6)$

3. $(x-2)(x-5)$

4. $(2x+5)(x+6)$

5. $(m^2+3)(5m+n)$

6. $(a^2+b^2)(a+b)$

7. $(x+5)(2x-3)$

8. $(x+5)(-3x+4)$

9. $(5x-3)(2x+7)$

10. $(x+4)(x^2+3x+5)$

11. $(3m+4)(m^2-3m+5)$

12. . $(2x-5)(4x^2-3x+1)$