## Algebra 1

## Lesson 8.3 - Factoring $a x^{2}+b x+c$, $\quad($ where $a=1)$ <br> Mrs. Snow, Instructor

Once upon a time..... back in chapter 7 ! we learned how to multiply 2 binomials to get a product of a trinomial. Let's take a closer look at what we did and how we can expand the application.

$$
\begin{gathered}
(x+2)(x+5)=x^{2}+7 x+10 \\
2+5=7 \\
2+5(2)(5)=10
\end{gathered}
$$

## 1. What do we note about the trinomial?

1. The constant term is the product of the constants in the binomial.
2. The sum of the constants in the binomial is the value of the linear term coefficient.

We can use this relationship to work backwards and factor a trinomial into its binomial factors.
Vocabulary:
Quadratic - a polynomial that can be written in the form $a x^{2}+b x+c$ where $a, b$, and $c$ are real numbers and $a \neq 0$.
$\boldsymbol{x}^{2}+\boldsymbol{b x}+\boldsymbol{c} \quad$ when " $c$ " is positive: find two numbers that will multiply to equal the constant term " c ", and add up to equal "b"

$$
x^{2}+10 x+24
$$

1. Make a list of all the factors of 24
2. Which factor pairs add up to equal the coefficient term?

| $=24$ | $=10$ |
| :---: | :---: |
| $1 \times 24$ | sum $=25$ |
| $2 \times 12$ | sum $=14$ |
| $3 \times 8$ | sum $=11$ |
| $\mathbf{4 \times 6}$ | $\mathbf{4 + 6 = 1 0}$ |

$$
\left(\begin{array}{lll}
x & )(x & )
\end{array}\right.
$$

$$
(x+4)(x+6)
$$

check:
(x+4)(x+6)
$x^{2}+6 x$
$+4 x+24=$
$x^{2}+10 x+24$
$\boldsymbol{x}^{2}+\boldsymbol{b x}-\boldsymbol{c}$ when " $\mathbf{c}$ " is negative: find two numbers that will multiply to equal " $-\boldsymbol{c}$ " but when subtracted will equal "b"

| $x^{2}+7 x-18$ |  |
| :---: | :---: |
| $=-18$ | $=7$ |
| $1 \times 18$ | diff $=17$ |
| $\mathbf{2 \times 9}$ | $\mathbf{9 - 2}=\mathbf{7}$ |
| $3 \times 6$ | diff $=3$ |

To have a positive 7 we will make 2 negative:

$$
(x+\underset{(x+9)(x-2)}{ })(x-\quad)
$$

Check:

$$
\begin{aligned}
& x^{2}-2 x \\
& \quad+9 x-18= \\
& x^{2}+7 x-18
\end{aligned}
$$

1. Make a list of all the factors of 18. Recognize that we will be looking at a positive factor and a negative factor! $(-) \times(+)=(-)$.
2. Which factor pair has the difference of 7 ? Then place signs such that the difference is positive 7 !
3. Now make a "template" of 2 sets of parentheses.
4. Recognize that the first term of each binomial will be an $\mathbf{x}$.
5. Now you can fill in the constant terms with +and-signs inserting the factor pairs such that the 2 values that multiply out to - 18 and have a difference of +7 .

What if $b$ is negative and $c$ is positive? ??: $x^{2}-b x+c$
$x^{2}+2 x-15 \quad x^{2}-6 x+8$
or!! $\boldsymbol{x}^{2}-\boldsymbol{b x}-\boldsymbol{c}$

$$
x^{2}-8 x-20
$$

Factoring Flow Chart for: $1 x^{2}+b x+c$

$$
\begin{array}{cc}
+\boldsymbol{b} \\
(+)(+) & \begin{array}{c}
+\boldsymbol{b} \\
(-)(-)
\end{array} \\
\begin{array}{c}
x^{2}+3 x+2 \\
(x+1)(x+2)
\end{array} & \begin{array}{c}
x^{2}-6 x+5 \\
(x-1)(x-5)
\end{array}
\end{array}
$$

| $+\boldsymbol{b}$ | $\underbrace{}_{-\boldsymbol{b}}$ |
| :---: | :---: |
| $(+$ big $)(-$ small $)$ | $(-$ big $)(+$ small $)$ |
| $x^{2}+7 x-18$ | $x^{2}-5 x-14$ |
| $(x-2)(x+9)$ | $(x-7)(x+2)$ |

To show that a quadratic and its factored form are the same, you can select values for your variable and evaluate:
Is $(n+3)(n+8) \stackrel{?}{=} n^{2}+11 n+24$; check for $n=0,1,2,3,4$

$$
(0+3)(0+8)=0^{2}+11(0) n+24
$$

$$
24=24
$$

Now you try for the remaining values of $n$.

