

**Algebra I**  
**Lesson 12.1 – Inverse Variation**  
**Mrs. Snow, Instructor**

In chapter 5 we learned that when two variable quantities have a constant (unchanged) ratio, their relationship is called a **direct variation**. We say that  $y$  varies directly as  $x$ . The constant ratio,  $k$ , is called the **constant of variation**. The formula for direct variation is:  $y = kx$  where:  $k = \frac{y}{x}$ . Basically, our constant of variation is our slope.

There is another relationship between  $x$  and  $y$  that is known as **Inverse Variation**. In an inverse variation, the values of the two variables change in an opposite manner, that is, as one value increases, the other proportionately decreases. We say that  $y$  varies inversely to  $x$  or  $y$  is inversely proportional to  $x$ .

**Vocabulary**

**Inverse Variation** – A relationship that can be written in the form  $y = \frac{k}{x}$ , or  $x = \frac{k}{y}$ , where  $k = xy$

**Direct Variation** – A relationship that can be written in the form  $y = kx$ , where  $k = \frac{y}{x}$

**Constant of Variation** – is the number that relates two variables that are directly proportional or inversely proportional to one another. The constant  $k$  in direct and inverse variation equations.

Question: Is the graph of an inverse variation linear? What is the form of a linear equation?

How can we tell if a set of data is direct or inverse variation? Well, what is our constant of variation equal to for our different variation problems?

<b>Direct variation</b>	<b>Inverse variation</b>
look for a constant rate of change: $k = \frac{y}{x}$	look for a constant product: $k = xy$

Are the relationships direct or inverse variations? Write the equation that models the data.

<table border="1" style="width: 100%; text-align: center;"> <tr><td><b>x</b></td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td><b>y</b></td><td>2.5</td><td>5</td><td>7.5</td><td>10</td></tr> </table>  <table border="1" style="width: 100%; text-align: center;"> <tr><td><b>x</b></td><td>2</td><td>4</td><td>6</td></tr> <tr><td><b>y</b></td><td>3.2</td><td>1.6</td><td>1.1</td></tr> </table>	<b>x</b>	1	2	3	4	<b>y</b>	2.5	5	7.5	10	<b>x</b>	2	4	6	<b>y</b>	3.2	1.6	1.1	<table border="1" style="width: 100%; text-align: center;"> <tr><td><b>x</b></td><td>-2</td><td>-1</td><td>1</td><td>5</td></tr> <tr><td><b>y</b></td><td>-5</td><td>-4</td><td>4</td><td>.8</td></tr> </table>	<b>x</b>	-2	-1	1	5	<b>y</b>	-5	-4	4	.8	<table border="1" style="width: 100%; text-align: center;"> <tr><td><b>x</b></td><td>1</td><td>2</td><td>4</td><td>5</td></tr> <tr><td><b>y</b></td><td>-2.5</td><td>-1.25</td><td>-.625</td><td>-.5</td></tr> </table>	<b>x</b>	1	2	4	5	<b>y</b>	-2.5	-1.25	-.625	-.5
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When given an equation and asked to determine if it is an inverse relationship, see if the equation can be written in the form  $y = \frac{k}{x}$ .

Which are inverse variation equations?

-7xy = 49	2x + y = 8	3y = 3x - 2y	4xy + 5x = 6 + 5x
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**Example:** Given  $y$  varies inversely as  $x$ . Write a variation function when  $y = 1.4$  and  $x = 0.3$ .

What is the value of  $y$ , when  $x = 5$ ?  
 $x$  when  $y = -0.3$ ?

- Using our equation for inverse variation, substitute the values for  $x$  and  $y$ . Solve for  $k$ .
- With the value of  $k$ , write the equation for inverse variation.
- You now have an equation that for any given value of  $x$  you can find  $y$  and visa versa.

$x$  and  $y$  vary inversely. When  $x = 16, y = 0.5$ . Write an equation that models this relationship.

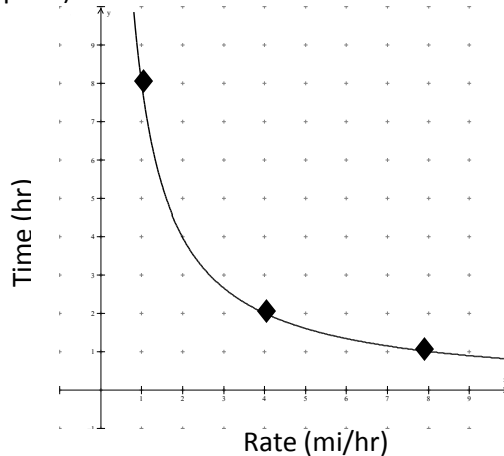
What is  $x$  when  $y = 30$ ? What is  $y$  when  $x = -2$ ?

Let's look at an example. How long will it take a cyclist to bike 8 miles? Well that depends on his speed. A biker traveling at 8 mph can cover 8 miles in 1 hour. If the biker's speed decreases to 4 mph, it will take the biker 2 hours to cover the same distance.

Rate (mi/hr)	Time (hr)
8	1
4	2
1	8

Notice that as the rate decreases, the time increases. Cut the rate in half, the time doubles. Our rate equation may be written as  $t = \frac{d}{r}$ , at distance equal to 8 miles we get:  $t = \frac{8}{r}$  or  $tr = 8$

Graphically we see can see the relation between time and rate (speed).



Graph:  $y = \frac{6}{x}$ ,  $y = -\frac{2}{x}$ , and  $y = \frac{1}{x}$

Yes! Table of values, choose both negative and positive values for  $x$ . You

$x$	$y$
-8	
$-\frac{1}{2}$	
0	
2	
8	

$x$	$y$

An inverse variation graph is made up of 2 parts or branches. Note the graph of an inverse variation will never contain the point  $(0, 0)$ . Why?

Well, look at the equation, can  $x=0$ ?  
 There are **Asymptotes** at  $x = 0$  and  $y = 0$

