**Algebra I**

Lesson 11.3 – Exponential Growth and Decay

**Mrs. Snow, Instructor**

**Exponential functions** are used to predict population growth patterns, nuclear power reactions, ozone, and rain forest depletion. There are two types of exponential functions: **growth** function and **decay** function.

**Vocabulary**

**Growth function –** over a period of time the output (y) will grow. As **x** increases **y** will increase.

**Decay function –** over a period of time the output (y) will decay, get smaller. As **x** increases, **y** will decrease.

**Exponential function –** $y = ab^x$

**Parent function –** exponential function in the simplified form $y = b^x$

**Growth factor – b;** the base number that is raised to the power of x is called the growth factor. It is related to the rate of growth or decay and dependent upon the value of **r.** $\boldsymbol{b = 1 \pm r}$

**Rate of increase – r;** the constant rate increase or decrease generally stated as a % but calculated as a decimal.

**Half-life –** the time it takes for ½ a substance to decay into another substance.

**Compound interest –** interest earned or paid on both the principal and previously earned interest.

The exponential form $y = ab^x$ can be written to calculate growth or decay patterns, given a fixed rate of increase (growth) or decrease (decay). The base is: $b = 1 \pm r$, where **r** is the rate of growth or decay.

$$\boldsymbol{A(t) = a(1 \pm r)^t}$$

where A(t) is the final amount

**a** is the initial amount

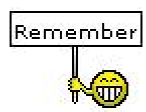
**r** is the rate of increase $(1 + r)$

or decrease $(1 - r)$

**t** is the number of time periods

a growth rate of 15% would be $A(t) = a(1.15)^t$

a decrease of 15% would be $A(t) = a(.85)^t$




growth shows a base greater than 1 and decay has a base greater than 0 and less than 1

So, by looking at these equations, which show exponential growth and which show decay?

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|---|---|---|---|
| $y = 72(1.6)^x$<br>$b > 1? \quad or \quad 0 < b < 1?$ | $y = 24(0.8)^x$ | $y = 3\left(\frac{6}{5}\right)^x$ | $y = 7\left(\frac{2}{3}\right)^x$ |

This basic formula may be revised to calculate compound interest, bacteria colony growth, human population patterns, appreciation (increased value) and depreciation (decreased value) of cars, trucks, homes, or office buildings, and many other similar applications.

So let's take a look an example. A new car costs about $24,500. It is estimated that the car will depreciate (lose value) by 15% each year. What will the car be worth in 4 years?

Since the car is losing value, we will need to use the formula, $A(t) = a(1 - r)^t$, where a=$24,500, r=15%, and t=4 years. Thus, A$(t) = 24500(1 - 0.15)^4$

$= 24500(0.85)^4$

$[24500 \times 0.85 \ \wedge 4\, ENTER]$

$\approx \$12{,}789.15$ This means that the car will lose about half if its value in the first 4 years!

A sculpture is increasing in value at a rate of 8% per year, and its value in 2006 was $1200. Write an exponential function to model this situation. Then find the sculpture's value in 2012.
Growth or decay?
Rate? %=________
b=______________
initial amount?____________

In a particular region of a national park there are currently 400 deer. The population is decreasing at an annual rate of 11%. Write an exponential function to model the deer population. How many deer will be in the region in 5 years?
Growth or decay?
Rate? %=________
b=______________
initial amount?_____________

**Half-life** is the time it takes for half of a material to decay. Our exponential equation now becomes:

$$\boldsymbol{A = A_0 \frac{1^{\frac{t}{k}}}{2}}$$

**A** = amount of remaining material
**$A_0$** = initial amount
**½** = the rate of decay
**t**=time;
**k**=half-life time
Understand that your rate of decay is 50% so $b = (1 - r) = (1 - .5) = .5 = \frac{1}{2}$

The $\boldsymbol{\frac{t}{k}}$ is a bit tricky. You need to divide the half-life time into the decay time to figure out how many "half-life" units are in the total time.

If the half-life of a certain substance is 5 years, which means in 5 years only half of the original material will be remaining. What are the values for the exponent $\boldsymbol{\frac{t}{k}}$ if the decay time is 10 year?. $\boldsymbol{\frac{t}{k} = \frac{10}{5} = 2.}$

The half life of a certain pesticide is 12 days. What is the value of the exponent $\boldsymbol{\frac{t}{k}}$ for a time of 30 days?

The half-life of carbon-14 is 5700 years. Write an exponential function to model of the decay of a 410-mg sample. How much carbon-14 is remaining after 4514 years?

**Compound Interest:** Banks have many different ways to calculate interest on both loans and investments. Compound interest is a calculation where interested is computed on both the principal and previously earned interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

**A=** the balance after **t** years

**P=** the principal or original amount

**r=** the annual interest rate expressed as a decimal

**n=** the number of times interest is compounded per year

**t=** time in years

Write a compound interest function to model each situation. Then find the balance after the given number of years.

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|---|
| $4000 is invested at a rate of 3.5% compounded quarterly. What is the investment after 8 years |
| $4000 is invested at a rate of 3.5% compounded monthly. What is the investment after 8 years? |
| Which is a better investment? Compounding quarterly or monthly? |