

## Algebra II

### Lesson 5: Inverse Functions

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Many actions are reversible. A light can be turned on and a light can be turned off. In mathematics this concept of reversing a calculation and arriving at the original result is associated with an inverse. A function can be described as a "DO" and the inverse can be described as the "UNDO." Put another way, an inverse relation is an exact opposite of what a function does and has a special symbol;  $f^{-1}(x)$ .

Write an equation to describe the operations:

Subtract 7 from  $x$  and divide the result by 2

$$1 \xrightarrow{\quad} 2$$

$$\textcircled{2} \xleftarrow{\quad} \textcircled{1} \quad \text{Inverse}$$

$$y = \frac{x-7}{2}$$

Now, state the inverse actions. This is performed by stating the reverse order and applying the inverse operation at each step.

$\div$  inverse  $\times$  ;  $-$  inverse  $+$   
multiply by 2 & add 7  
 $y = 2x + 7$

There are 2 basic steps to formulating an inverse relation.

- Step 1 Switch the  $x$  and  $y$  in the equation
- Step 2 Solve for the new " $y$ ", and replace  $y$  with  $f^{-1}(x)$

**Example:** Find the inverse function for

$$y = x + 3$$

$$x = y + 3$$

$$x - 3 = y$$

$$f^{-1}(x) = x - 3$$

Step 1: switch  $x$  and  $y$

solve

Step 2: replace  $y$  with  $f^{-1}(x)$

Find the inverse of the function

Switch  $x$  &  $y$   $y = x^2 + 3$

$$x = y^2 + 3$$

$$x - 3 = y^2$$

$$\sqrt{x-3} = y$$

$$f^{-1}(x) = \pm \sqrt{x-3}$$

$y = 3x + 10$  switch spots

$$x = 3y + 10$$

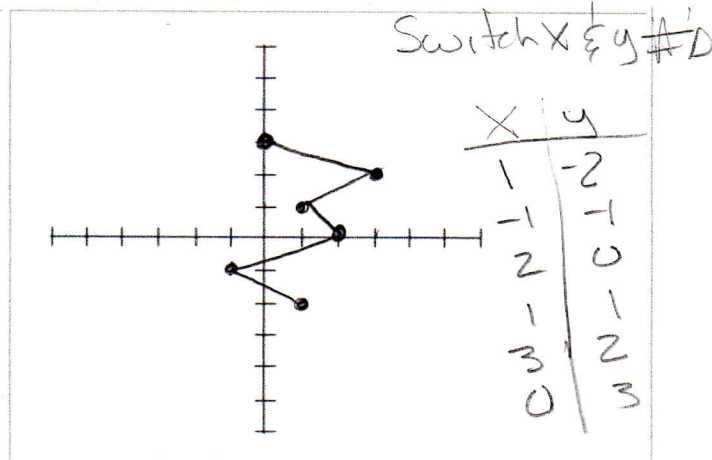
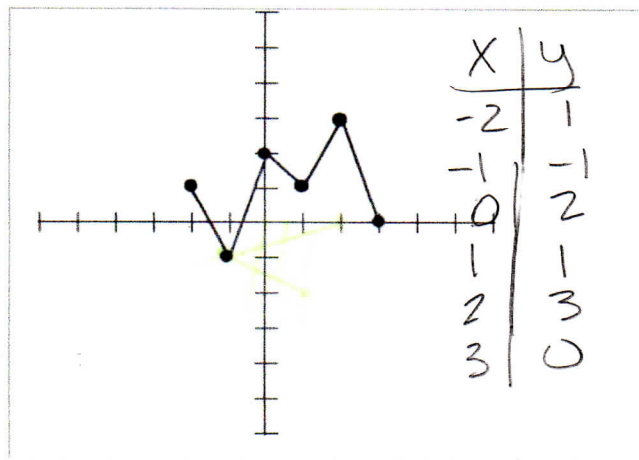
$$\frac{1}{3}(x-10) = y$$

$$\frac{x-10}{3} = y = f^{-1}(x)$$

**DOMAIN AND RANGE**

The domain of  $f$  equals the range of  $f^{-1}$ . The range of  $f$  equals the domain of  $f^{-1}$ . OH!! So when given a table of values or even a graph, to find the inverse switch the  $x$  and  $y$  values! When you are required to plot an inverse, remember, the inverse of a function has all the same points as the original function, except that the  $x$ 's and  $y$ 's have been switched. Example:  $f(x) = \{(1, 0), (-3, 5), (0, 4)\}$   $\therefore$   $f(x)^{-1} = \{(0, 1), (5, -3), (4, 0)\}$ . Switch the  $x$  and  $y$ -values.

Graph the inverse of the function  $f(x)$  graphed below. Is the inverse a function? Explain



For each of the following functions – 1) find the inverse, 2) find the domain and range of the inverse, 3) determine whether the inverse is a function, and 4) then evaluate (if given) for  $f^{-1}(-1)$ ,  $f(2)$ , and  $f^{-1}(3)$ .

$f^{-1}(x)$   
function  
no repeats  
 $f^{-1}(-1) = 0$   
 $f^{-1}(2) = 5$

x	f(x)
-2	1
-1	0
0	-1
1	-3
2	-5

x	f <sup>-1</sup> (x)
1	-2
-1	0
-3	1
-5	2

$f^{-1}(x)$   
D:  $\{-5, -3, -1, 0, 1\}$   
R:  $\{-2, -1, 0, 1, 2\}$

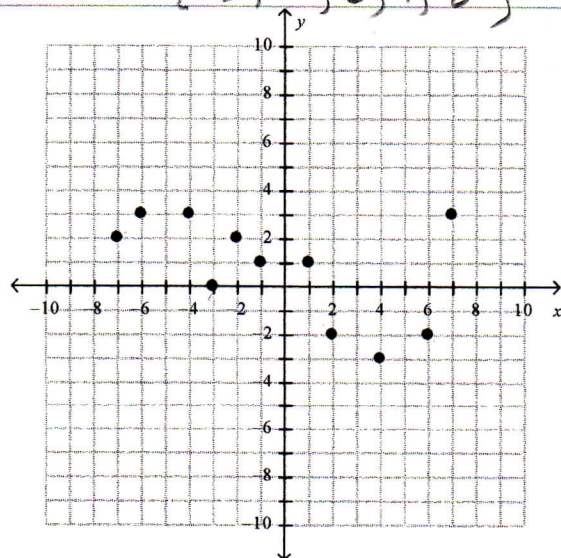
p	f(p)
-3	3
-2	6
1	3
2	0
3	-9

p	f <sup>-1</sup> (p)
3	-3
6	-2
3	1
0	2
-9	3

$f^{-1}(3) = -3$

$f^{-1}(p)$   
D:  $\{-9, 0, 3, 6\}$   
R:  $\{-3, -2, 1, 2, 3\}$

Not function  
repeats



x	f(x)
-7	2
-6	3
-4	3
-3	0
-2	2
-1	1
1	1
2	-2
4	-3
6	-2
7	3

x	f <sup>-1</sup> (x)
2	-7
3	-6
3	-4
0	-3
2	-2
1	-1
1	1
-2	2
-3	4
-2	6
3	7

$f^{-1}(x)$   
not function  
 $f^{-1}(2) = -7, -2$   
 $f^{-1}(3) = -6, -4, 7$

D  $f^{-1}(x)$   $\{-3, -2, 0, 1, 2, 3\}$

R  $f^{-1}(x)$   $\{-7, -6, -4, -3, -2, -1, 1, 2, 4, 6, 7\}$