## Algebra II

## Lesson 9-6: Solving Rational Equations

Mrs. Snow, Instructor
In this section we will learn techniques to solve for our variable when it is located in both the numerator and denominator. It is a fairly straight forward process, but the catch is that when multiplying an equation by an algebraic expression, there is a chance of getting extraneous solutions. So once again with the risk of extraneous expressions, we must check our solutions to verify that all are true solutions and/or the solution is not a restriction.

## Using cross-multiplication:

Solve:

$$
\frac{-4}{5(x+2)}=\frac{3}{x+2}
$$

1. identify restrictions on the variable
2. take the cross product
3. using the distributive property simplify
4. isolate $x$ terms and solve

Try:

## 1. Restrictions??

## Using LCD:

When the equation includes addition or subtraction of terms, look at all the denominators and find the LCD.
Multiply through by reciprocal of the LCD to clear out the denominators. Then proceed solving for x .

Solve:

$$
\begin{gathered}
\frac{4}{x}-\frac{3}{x+1}=1 \\
x \neq-1,0 \\
L C D=x(x+1) \\
x(x+1) \cdot\left(\frac{4}{x}-\cdot \frac{3}{x+1}\right)=x(x+1) \cdot 1 \\
x(x+1) \cdot\left(\frac{4}{x}\right)-x(x+1)\left(\frac{3}{x+1}\right)=x(x+1) \cdot 1 \\
4(x+1)-3 x=x(x+1) \\
4 x+4-3 x=x^{2}+x \\
4=x^{2} \\
x= \pm 2
\end{gathered}
$$

1. determine restrictions
2. find the LCD
3. multiply through by the LCD; what you do the left, you do to the right.
4. use distributive property to simplify
5. Isolate and solve for $x$.
6. verify that solutions are legit.
|ry: $\quad \frac{1}{2 x}-\frac{2}{5 x}=\frac{1}{2} \quad \frac{11}{3 x}-\frac{1}{3}=\frac{-4}{x^{2}}$

## Application Problems:

Solving rational equations frequently shows up in application problems. Let's look at a problem: Carlos can travel 40 miles on his motorbike in the same time it takes Paul to travel 15 miles on his bicycle. If Paul rides his bike $20 \mathrm{mi} / \mathrm{hr}$ slower than Carlos rides his motor bike, find the speed for each bike.

1. Write the facts about each individual: Carlos' speed $=c \quad$ Paul's speed $=p=c-20$
Carlos' distance=40 Paul's distance=15
2. What relationship is equal, that is, what is in common? Ans: time?
3. Since we are talking speed and time and distance, what is the equation can we use?

$$
s=\frac{d}{t}, \text { that is speed equals distance over time, solving for time: } t=\frac{d}{s}
$$

Since the times are equal we know that the ratio of $\frac{d}{s}$ for the boys is equal too:

$$
\begin{array}{cl}
\frac{40}{c}=\frac{15}{c-20} & \begin{array}{l}
\text { now set about to solve for } \mathrm{c}! \\
40(c-20)=15 c
\end{array} \\
40 c-800=15 c & \text { cross multiply } \\
25 c=800 & \\
c=32 \mathrm{mph} \text { Carlos } \mathrm{C} \text { and solve for } \mathrm{p} \\
\therefore \text { Paul's speed }=12 \mathrm{mph} &
\end{array}
$$

Now what about those "time to do a job" situations?
Jason can clean a large tank at an aquarium in 6 hours. When Jason and Lacy work together, they can clean the tank in 3.5 hours. How long would it take Lacy to clean the tank if she works by herself?

- Look at their rates: Jason's rate + Lacy's rate $=$ combined rate
- Jason's rate: 1 tank per 6 hours $\left(\frac{1}{6}\right)$
- Lacy's rate: 1 tank per $h$ hours $\left(\frac{1}{h}\right)$.
- the complete job rate is 1 tank per 3.5 hours
- substitute the rates into the rate equation: $\frac{1}{6}+\frac{1}{h}=\frac{1}{3.5}$
- solve for $h ; h=8.4$

Now if the facts were given that Jason could clean the tank in 6 hours, Lacy could clean the tank in 8.4 hours how long would it take if both worked together? It is the same basic set up:

$$
\text { Jason's rate }+L a c y^{\prime} \text { s rate }=\text { total rate } \therefore \frac{1}{6}+\frac{1}{8.4}=\frac{1}{t} \text { where } t=\text { total time }
$$

One pump can fill a tank with oil in 4 hours. A second pump can fill the same tank in 3 hours. If both pumps are used at the same time, how long will they take to fill the tank?

