## Algebra II

## Lesson 9.3: Rational Functions and Their Graphs

Mrs. Snow, Instructor
The functions studied in the previous sections are members of a bigger group of functions called rational functions. A rational function is a ratio of two polynomial functions: $f(x)=\frac{P(x)}{Q(x)}$. As seen with the reciprocal function, we will also have both vertical and horizontal asymptotes with rational functions.

## Vertical Asymptotes

When we look at a rational function realize there is a polynomial in the denominator. As we learned several chapters ago, the solutions to a polynomial are the values where $x=0$. Now, the moment we put a polynomial in a denominator of a fraction, we are going to get numbers that will make the denominator equal to zero, and we all know that is a bad thing to have. Values that make the denominator equal to zero are restrictions on the domain of the rational function. The restrictions on the domain are described by the following definitions:
discontinuous: a break in the line or curve on a graph
point of discontinuity: the point at $\mathbf{x}=\mathbf{a}$ where the function is undefined (point where the denominator $=0$ ).
vertical asymptote: the line formed by $\mathbf{x}=\mathbf{a}$, the curve will get closer and closer but never touch or cross this line.
How do we find the points of discontinuity and thus the vertical asymptotes? Set the denominator equal to zero and solve for x . These values are the points of discontinuity and hence the vertical asymptotes.

| Find the vertical asymptotes for: <br> $y=\frac{x+3}{x-4}$ | $y=\frac{x}{x^{2}-9}$ | $y=\frac{3}{x^{2}-x-12}$ |
| :--- | :--- | :--- |

Some rational functions may not have any restrictions while others may have one or more, depending on the denominator.

$y=\frac{2 x^{2}-2}{x^{2}-4}$ where $x \neq$
$2,-2$
$y=\frac{x^{2}+x-6}{x}$ where $x \neq 0$



Graph: $y=\frac{(x-2)(x+1)}{(x-2)}$
What does the graph look like?
Note that the ( $x-2$ ) cancels out in the numerator and denominator. Check the table generated by the graph what is the value of $y$ for $x=2$ ?

When $P(x)$ and $Q(x)$ have a common real zero at $a$, then there is a hole in the graph. (the graphing calculator may not show the hole)


## Horizontal asymptotes

The middle graph above shows a horizontal asymptote. Why is it at $y=2$ ? Note the following rules for horizontal asymptotes:

1. A rational function will have no more than one horizontal asymptote.
2. Degree of denominator is bigger than the degree of numerator: HA at $y=0$
3. Degree of numerator is bigger than degree of denominator: NO HA
4. Degree of numerator = degree of denominator: divide leading coefficients: $y=\frac{L C}{L C}$ And now for an advertisement of a new Sci Fi Movie!!

http://www.inmagine.com/izs015/izs015195-photo LOOK AT THE EXPONENTS TO DETERMINE THE HORIZONTAL ASYMPTOTE:

## BIGGER BIGGER EXPONENTS <br> ON <br> BOTTOM: <br> $\mathrm{O}=\mathrm{y}$

Describe the vertical asymptotes and/or holes of each rational function.

| $y=\frac{x-2}{x^{2}+2 x-3}$ | $y=\frac{2 x^{2}+5}{x^{2}-2 x}$ |
| :---: | :---: | :---: |
| $y=\frac{x-2}{x^{2}+x-6}$ | $y=\frac{3}{x+2}$ |
| $y=\frac{9-x^{2}}{x^{2}-9}$ | $y=\frac{6-3 x}{x^{2}-5 x+6}$ |

Find the horizontal asymptotes of the graph of each rational function

$$
y=\frac{3 x-4}{4 x+1} \quad y=\frac{2 x^{2}+5}{x+1} \quad y=\frac{x+3}{(x-1)(x-5)}
$$

When we know where and what type the points of discontinuity are, we can sketch graphs:


