Lesson 9-2
The Reciprocal Function Family

We all know what the reciprocal of a number is: one over the number, the reciprocal of \( a \) is \( \frac{1}{a} \). The reciprocal of \( 5 \) is \( \frac{1}{5} \). Well, functions like inverse variations are in a reciprocal form, hence we call these functions reciprocal functions. Generally speaking we will see the \( x \) variable in the denominator: \( f(x) = \frac{1}{x} \). Of course reciprocal functions can and will be more complicated so we need to be familiar with the complete form of a reciprocal function which is: \( f(x) = \frac{a}{x-h} + k \) (note the restriction on the domain of \( x \neq h \)). Here again are \( h \) and \( k \) which translate the parent function.

**Graphing**

Graph \( y = \frac{8}{x} \)
Identify vertical and horizontal asymptotes.

**Domain and Range**

1. Make a table of values that include both positive and negative values.
2. Graph the points and draw a line.
3. Asymptotes will be at domain restrictions, that is where \( x \) cannot be equal to zero is a vertical asymptote. No matter how big or small \( x \) becomes, \( y \) will never be equal to zero, hence, the horizontal asymptote.

   \[ This is a lot of points as you get more comfortable, you will be able to reduce the number, but you need at a minimum at least 3 points for each branch. \]

**Asymptotes**

Often we can see on the graph what the asymptotes are, but how can we look at the equation and determine the asymptotes? Let’s take a look at that general form of a reciprocal function; the “\( h \) and \( k \)” are significant: \( y = \frac{a}{x-h} + k \). The denominator cannot be equal to zero, so set it to zero and solve for \( x \). At \( x = h \) we have a vertical asymptote and at \( y = k \) we have a horizontal asymptote. So by putting the function into our general form, we can pluck off the asymptotes.

Also, if given a parent reciprocal function and asymptotes, we can develop an equation with the given asymptotes that will be a translation of the parent function.
Sketch the graph and identify the asymptotes

\[ y = \frac{6}{x} \]

\[ y = \frac{1}{x - 2} - 3 \]

\[ y = \frac{4}{x + 5} - 2 \]

\[ y = \frac{2}{x - 3} \]

Write an equation for the translation of \( y = \frac{7}{x} \)

Asymptotes are located at:

- \( x = 4 \) and \( y = -1 \)
- \( x = -1 \) and \( y = 3 \)