Lesson 9-1
Inverse Variation

When the ratio of two variables has a constant (unchanged) ratio, their relationship is called a **direct variation**. We say that \( y \) varies directly as \( x \). The constant ratio, \( k \), is called the **constant of variation**.

\[
\frac{y}{x} = k, \quad \text{or} \quad y = kx
\]

Note: in a linear situation the constant of variation is our slope:

\[
\frac{\text{rise}}{\text{run}} = \frac{y}{x}
\]

In direct variation problems, we will see that as one variable increases the other increases. Likewise as one decreases so will the other decrease.

Melissa’s weekly salary, \( s \), varies directly as the number of hours, \( h \), that she works. Write an equation that describes this relation. Solve for the constant of variation.

Melissa’s check shows she worked 32 hours and it is the amount of $363.20. What is her hourly rate?

According to Hooke’s Law, the force needed to stretch a spring is proportional to the amount the spring is stretched. If fifty pounds of force stretches a spring five inches, how much will the spring be stretched by a force of 120 pounds?

If \( y \) varies directly as \( x^2 \) and \( y = 8 \) when \( x = 2 \), find \( y \) when \( x = 1 \). Write the equation of variation.
The opposite of direct variation is known as **Inverse Variation**. In an inverse variation, the values of the two variables change in an opposite manner, that is, as one value increases, the other decreases.

\[ xy = k, \quad \text{so} \quad y = \frac{k}{x}, \quad \text{or} \quad x = \frac{k}{y} \]

Let’s look at an example. How long will it take a cycler to bike 8 miles? Well that depends on his speed. A biker traveling at 8 mph can cover 8 miles in 1 hour. If the biker’s speed decreases to 4 mph, it will take the biker 2 hours to cover the same distance.

<table>
<thead>
<tr>
<th>Rate (mi/hr)</th>
<th>Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Notice that as the rate decreases, the time increases. Cut the rate in half, the time doubles. Our rate equation may be written as \( t = \frac{d}{r} \), at distance equal to 8 miles we get: \( t = \frac{8}{r} \).

Graphically we see can see the relation between time and rate (speed).

Given \( y \) varies inversely as \( x \). Write a variation function when \( y = 1.4 \) and \( x = 0.3 \)

So: \( k =? \) and our inverse variation function is?

Determine if the relationship between the values is direct variation, inverse variation or neither. Write an equation if possible.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3.2</td>
<td>1.6</td>
<td>1.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.9</td>
<td>1.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>9.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>14</td>
<td>35</td>
<td>56</td>
<td>66.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>2.5</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>30</td>
<td>24</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>
Write the function that models each inverse variation. Then find \( y \) when \( x = 9 \).

\[
x = 3 \text{ when } y = -5
\]

The inverse variation contains the ordered pair: \((6, 3)\)

**Combined Variation**

Variation is not only for linear relationships. We can just as easily have a situation where \( y \) varies inversely with \( x^2 \), such that: \( y = \frac{k}{x^2} \). Also, we can have situations where we have what is called a **joint variation**. Here a variable will vary jointly with two other variables: \( z = kxy \). Let’s put some of these combinations into table form:

<table>
<thead>
<tr>
<th>Combined Variation</th>
<th>Equation form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z ) varies jointly with ( x ) and ( y )</td>
<td>( z = kxy )</td>
</tr>
<tr>
<td>( z ) varies jointly with ( x ) and ( y ) and inversely with ( w )</td>
<td>( z = \frac{kxy}{w} )</td>
</tr>
<tr>
<td>( z ) varies directly with ( x ) and inversely with the product ( wy )</td>
<td>( z = \frac{kx}{wy} )</td>
</tr>
</tbody>
</table>

Given that \( z \) varies directly with \( x \) and inversely with \( y \). Write a variation function when \( x = 6, y = 2, \text{ and } z = 15 \)

Given that \( z \) varies jointly with \( x \) and \( y \). Write a variation function when \( x = 2, y = 3 \text{ and } z = 60 \)

Describe the combined variation that is modeled by each formula:

\[a) \quad A = \pi r^2 \]
\[b) \quad h = \frac{2A}{b} \]
The volume $V$ of a tetrahedron varies jointly with its altitude $h$ and base of area $b$. Find the formula that models this joint variation. Given that the tetrahedron has an altitude of 5 cm, a base area of 6 cm², and a volume of 10 cm³.

Other stuff

Given a direct variation, find the missing variable for the pair of values: $(4,6), (x, 3)$

Given an inverse variation, find the missing variable for the pair of values: $(4,6), (x, 3)$