Lesson 8-4 Properties of Logarithms
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Last section we saw that $pH = -log\,[H+]$ can also be expressed in exponential form: $10^{-pH} = [H^+]$. Since logarithms are inverses of exponents, you can derive the properties of logarithms from the properties of exponents:

Operation	**Logarithms** ***NEW***	***Example***
product	$log_b(m \cdot n) = log_b m + log_b n$	$\log_3 8 \cdot 4 = \log_3 8 + \log_3 4$
quotient	$log_b\, {}^{m}/_{n} = log_b m - log_b n$	$\log {}^{9}/_{11} = \log 9 - \log 11$
power	$log_b m^x = x\,log_b m$	$\log_2 x^7 = 7\log_2 x$

Example: express as a single logarithm

$log7 + log2$

$\log(7 \cdot 2) = \log 14$

$log_2 12 - log_2 3$

$\log_2 \frac{12}{3} = \log_2 4$

$log_3 8 - 2log_3 6 + log_3 3$

$= \log_3 8 - \log_3 6^2 + \log_3 3 =$

$= \log_3 \frac{8}{36} + \log_3 3 =$

$\log_3 \frac{\cancel{8}^{2}}{\cancel{36}_{\cancel{12}\,3}}(\cancel{3}) = \log_3 {}^{2}/_{3}$

$ln5 - xln2$

$\ln 5 - \ln 2^x =$

$\ln \frac{5}{2^x}$

We can write as single logarithms and we can expand into multiple logarithms:

$log_8 x^3 y^5 =$

$\log_8 x^3 + \log_8 y^5 =$

$3\log_8 x + 5\log_8 y$

$log8\sqrt{x}$

$\sqrt{x} = x^{1/2}$

$\log 8 + \log x^{1/2}$

$\log 8 + \frac{1}{2}\log x$

$ln(7x)^3 =$

$3\ln 7x =$

$3[\ln 7 + \ln x] =$

$3\ln 7 + 3\ln x$

$log_m 25x^4$

$\log_m 25 + \log_m x^4$

$\log_m 25 + 4\log_m x$

Properties of logarithms may be applied and then the single logarithm may be evaluated:

Simplify: $3log_2 2 - log_2 4 = x$ $\log_2 2^3 - \log_2 4 = x$ $\log_2 8 - \log_2 4 = x$ $\log_2 8/4 = x$ $\log_2 2 = x \Rightarrow 2^x = 2^1$ $x = 1$	1. apply product rule or 2. quotient rule 3. simplify 4. evaluate: ____ $= x$ *You must read and understand the directions. Depending on what is required, you will either stop at this point or continue on to evaluate, that is x=....* $3\log_2 2 - \log_2 4 = 1$
$6 = log_3 3 + 5log_3 3 = x$ $\log_3 3 + \log_3 3^5 = \log_3 (3)(3^5)$ $= \log_3 3^6 = x$ $3^x = 3^6$ $x = 6$	$6lne = x = 6$ $\ln e^6 = x$ $\log_e e^6 = x$ ⇐ remember rule? $e^x = e^6$ $x = 6$

Logarithms are uses to model sound. The intensity of a sound is the measure of the energy carried by the sound wave. The greater the intensity of a sound, the louder it seems. Loudness is measured in decibels with the formula: $L = \mathbf{10log}\frac{I}{I_o}$. (I is the intensity of the sound in watts per square meter and I_o is the lowest intensity sound that the average human can hear.)

Earplugs are advertised to block a certain amount of noise. One earplug brand claims to block the sound of noise as loud as 22 dB. A second brand claims to block 8 times that amount. If this claim is true, how many more decibels are blocked? (subtract)

First off this is a subtraction problem as we are looking at "how many more." So let L_2= brand 2 loudness and

L_1= brand 1 loudness. Identify the relationship between the two brands: $I_2 = 8I_1$, so using our equation for loudness:

L_1 = Brand 1 $\quad L_2$ = Brand 2

$$L_1 = 10\log\frac{I_1}{I_o}$$

$$L_2 = 10\log\frac{I_2}{I_1} = 10\log\frac{8I_1}{I_o}$$

$$I_2 = 8I_1$$

Difference $L_2 - L_1 =$

$$= 10\log\frac{8I_1}{I_o} - 10\log\frac{I_1}{I_o}$$

$$= 10\left(\log 8 + \log\frac{I_1}{I_o}\right) - 10\log\frac{I_1}{I_o}$$

$$= 10\log 8 + \cancel{10\log\frac{I_1}{I_o}} - \cancel{10\log\frac{I_1}{I_o}}$$

$$= 10\log 8 \approx 9\text{ dB}$$