Lesson 8-4 Properties of Logarithms  
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Last section we saw that \( \text{pH} = -\log [H^+] \) can also be expressed in exponential form: 
\( 10^{-\text{pH}} = [H^+] \). Since logarithms are inverses of exponents, you can derive the properties of logarithms from the properties of exponents:

<table>
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<tr>
<th>Operation</th>
<th>Logarithms NEW</th>
<th>Example</th>
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<tr>
<td>product</td>
<td>( \log_b (m \cdot n) = \log_b m + \log_b n )</td>
<td>( \log_3 8 \cdot 4 = \log_3 8 + \log_3 4 )</td>
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<tr>
<td>quotient</td>
<td>( \log_b \frac{m}{n} = \log_b m - \log_b n )</td>
<td>( \log_9 \frac{9}{1} = \log_9 9 - \log_9 1 )</td>
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<tr>
<td>power</td>
<td>( \log_b m^x = x \log_b m )</td>
<td>( \log_2 \chi^7 = 7 \log_2 \chi )</td>
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Example: express as a single logarithm

\[
\log_7 7 + \log_2 2 \\
\log_3 8 - 2\log_3 6 + \log_3 3 \\
\ln 5 - \ln x^x \\
\log_x \sqrt{x} \\
\log_m 25x^4
\]

We can write as single logarithms and we can expand into multiple logarithms:
Properties of logarithms may be applied and then the single logarithm may be evaluated:

| Simplify: | 1. apply product rule  
| log₂ 2³ - log₂ 4 = x  
| log₂ 8 - log₂ 4 = x  
| log₄ 16 = x  
| log₆ 3 = x  
| 2³ = x  
| 2² = 2² | 2. quotient rule  
| 3 log₂ 2 - log₂ 4 = 1  
| 3 log₆ 3 + 5 log₆ 3 = log₆ (3³·3⁵)  
| log₆ 3³ = x  
| 3³ = 3³  
| x = 6 | 3. simplify  
| 4. evaluate: ___ = x  
| You must read and understand the directions.  
| Depending on what is required, you will either stop at this point or continue on to evaluate, that is x=...  
| 6ln e = x  
| ln e = x  
| x = e  
| x = 6 |
Logarithms are used to model sound. The intensity of a sound is the measure of the energy carried by the sound wave. The greater the intensity of a sound, the louder it seems. Loudness is measured in decibels with the formula: \( L = 10 \log \frac{I}{I_o} \). (I is the intensity of the sound in watts per square meter and \( I_o \) is the lowest intensity sound that the average human can hear.)

Earplugs are advertised to block a certain amount of noise. One earplug brand claims to block the sound of noise as loud as 22 dB. A second brand claims to block 8 times that amount. If this claim is true, how many more decibels are blocked? First off this is a subtraction problem as we are looking at “how many more.” So let \( L_2 = \) brand 2 loudness and \( L_1 = \) brand 1 loudness. Identify the relationship between the two brands: \( I_2 = 8I_1 \), so using our equation for loudness:

\[
\begin{align*}
L_1 &= \text{Brand 1} = 10 \log \frac{I_1}{I_o} \\
L_2 &= \text{Brand 2} = 10 \log \frac{I_2}{I_o} = 10 \log \frac{8I_1}{I_o}
\end{align*}
\]

\[
\text{Difference } L_2 - L_1 = 10 \log \frac{8I_1}{I_o} - 10 \log \frac{I_1}{I_o} = 10 \left( \log 8 + \log \frac{I_1}{I_o} \right) - 10 \log \frac{I_1}{I_o} = 10 \log 8 \approx 9 \text{ dB}
\]