## Algebra II

## Lesson 8-5: Exponential and Logarithmic Equations <br> Mrs. Snow, Instructor

We have been looking at exponential equations and techniques to evaluate them, that is solve for $x$. We found that in a situation where for example: $3^{x}=243$ we were able to get the right side into an exponential form and we found that $3^{x}=3^{5}$. We also know a rule that says if the bases are equal the exponents must be equal; therefore $x=5$.

Solve: $\quad 2^{x}=30$

Plan A: In order to solve this equation, we will need $x$ to be down on the ground floor. This can be done with logarithms, so we get: $\log _{2} 30=x$. Here we have the problem that we don't have the where with all to solve a base $2 \log !!$
Plan B: Golden rule of algebra: What you do to the left do to the right. This combined with our power property of logarithms will allow us to solve for $x$. So let's look back at our equation:

$\left.2^{x}=30 \quad \left\lvert\, \begin{array}{l}\text { 1. } \begin{array}{l}\text { Take the log of both sides } \\ \text { 2. With the power property, take the } \\ \text { exponent down in front of the log as a } \\ \text { coefficient } \\ \text { Divide each side by the } \log 2\end{array} \\ \text { Use a calculator to simplify }\end{array}\right.\right]$
$e^{2 x+1}=37$

When $\mathbf{x}$ is part of the argument of a log function, a similar process may be used:

## Solve:

$$
\log (3 x+1)=5
$$

1. Already in log form so make into an exponential function (base 10). Recognize that by rewriting into exponential form, we can unlock the $x$ and get it out of the log!!
2. solve for $\mathbf{x}$ by isolating the $\mathbf{x}$ term and by clearing coefficient
$2 \log x=-1$
$2 \log x-\log 3=2$

| $\log _{2}(6 x)-3=-2$ | $\ln (3 x)=6$ |
| :--- | :--- |

## CHANGE OF BASE

Now let's take a look at logs with bases other than 10. Evaluate $\boldsymbol{\operatorname { l o g }}_{3} \mathbf{2 9}$, there is no neat solution for this equation, unless we come up with another method. A method that works for these types of equations is called the Change of Base Formula.

$$
\text { Argument } \log _{b} M=\frac{\log _{c} M}{\log _{c} b}
$$

- the argument is kept in the numerator above the base in the denominator! Or, the base stays in the basement! Whew!

Letting c be our base 10, we may take a ratio of the log of the argument to the log of the original base. So our
equation above becomes: $\quad \log _{3} 29=\frac{\log 29}{\log 3}=3.065$

| $\log _{2} 7$ | $\log _{3} 54$ |
| :--- | :--- |
|  |  |

