**Algebra II**
**Lesson 8-3: Logarithmic Functions as Inverses**
**Mrs. Snow, Instructor**

Logarithms are used to express very large numbers or very small numbers in a more manageable way. The common logarithm, base 10, is used in science and engineering applications. Base 2 is used in computer science applications and base e, the natural logarithm, is used in the natural sciences and in pure mathematics such as calculus. Remember our problems where we had to guess and check to figure out the numbers of years it would take to save a certain amount of money? Well, a logarithm will undo that exponent and allow us to easily and quickly solve for the exponent.

Recall that each algebraic operation has an opposite or an inverse:

| **Operation** | **Inverse** |
|---|---|
| multiplication | division |
| addition | subtraction |
| square: $x^2$ | square root: $\sqrt{\ }$ |
| cube: $x^3$ | cube root: $\sqrt[3]{\ }$ |

Logarithms are the "opposite" (inverse) of exponents, just as subtraction is the opposite of addition. Logs "undo" exponentials. Technically speaking, logs are the inverses of exponentials. More practically speaking, you can think of logs in terms of the relationship:

$log_b(y) = x$ ← is equivalent to → $y = b^x$

base    base

*We say this as the "log-base-b of y equals x";*

| **Logarithm** | **Exponential** | **To change forms** |
|---|---|---|
| b is the base<br>$b > 0$ $and$ $b \neq 1$<br>inside of the log is the **argument** | b is the base<br>$b > 0$<br>$and$ $b \neq 1$ | $log_b x = y$ $so$ $b^y = x$ |

To change forms always start with the base. **The base is the base regardless of it being a log or an exponential.**

Since exponential and logs are inverses, when we have equation in one form, we can write it in the other form. The **common logarithm** is base 10 and unless otherwise written, log is understood to be base 10. So: $log_{10}y = \log y$.

| | |
|---|---|
| **Write** $10^2 = 100$ in logarithmic form<br><br>$10^2 = 100$ $and$ $\boldsymbol{log_{10}100 = 2}$ | $49 = 7^2$ |
| $10^{-1} = \dfrac{1}{10}$ | $729 = 3^6$ |

**If $2^x = 2^4$.** $\quad$ ***what does x equal??????***

We can also go from log to exponent thus solving for **x**:

| | |
|---|---|
| Evaluate $\quad log_8 16 = x$ | 1. convert to exponential form<br>2. write each side as 2 to a power<br>3. power property of exponents<br>4. set the exponents equal to each other<br>5. solve for x |
| $log_4 2 = x$ | $log_4 4 = x$ |
| $log_6 1 = x$ | $log_{10} 0 = x$ |

**The last examples show us that:**

| | | |
|---|---|---|
| $log_b b = 1$ | $log_b 1 = 0$ | $log_b 0$ *is undefined for all base* |

**Natural Logarithm:** The inverse of the function $\boldsymbol{y = e^x}$ **is** $\boldsymbol{log_e y = x}$ ***or*** $\boldsymbol{ln\, y = x}$ **.**
**remember:** $\boldsymbol{log_e = ln}$
When a relationships in the real world needs to be described with a logarithmic function, they are best described using the natural logarithms.

| | |
|---|---|
| Evaluate: $ln\, e^6$<br><br>Write in exponential form: | Write in exponential form:<br>$7.389 = e^x$ |
| Write in exponential form:<br>$ln\, 5 = 1.609$ | |

**Chemistry** *While in this section, we will cover pH at the end of this chapter*
pH of a substance is the measure of acidity of a liquid which is related to the hydrogen ion concentration ( $[h^+]$ ) of the liquid. A liquid becomes more acidic as the number of hydrogen ions increases. A logarithmic equation relates pH to concentration hydrogen ions:

$$\boldsymbol{pH = -log[h^+]}$$

**Optional:**

| | |
|---|---|
| **Find the concentration of hydrogen ions, $[H^+]$**<br>lime juice pH = 2.2<br>$2.2 = -logH^+$ | • log is understood to be base 10<br>• rewrite with base 10 written and move negative to other side.<br>convert into exponential form solve |
| cider vinegar pH=3.1 | egg white pH=8.0 |

Find the **inverse** of each logarithmic function:

1. Write in exponential form,
2. Switch: x is y and y is x.

| $log_5 x = y$ | $log_2(x+3) = y$ | $log_3(x+1) - 2 = y$ | $y = \ln x + 7$ |
|---|---|---|---|
| | | | |

Since we really don't have the capabilities to calculate y values for a log base b, we need to look at the inverse of a log function. The inverse of a log is an exponential function.

**Graph**: $log_5 x = y$

1. $rewrite\ in\ exponential\ form$:

2. By the definition of logarithm, the log is the inverse (x is y and y is x).

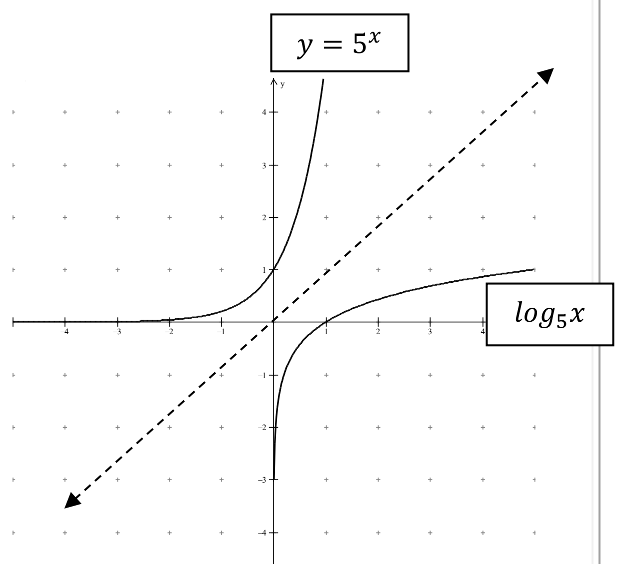
3. graph the exponential function

$y = 5^x$ **exponential:** $log_5 x = y$ **log:**

| x | y | **x** | **y** |
|---|---|---|---|
| -2 | | | -2 |
| -1 | | | -1 |
| 0 | | | 0 |
| 1 | | | 1 |
| 2 | | | 2 |

*(reflection across line of* $y = x$*)*

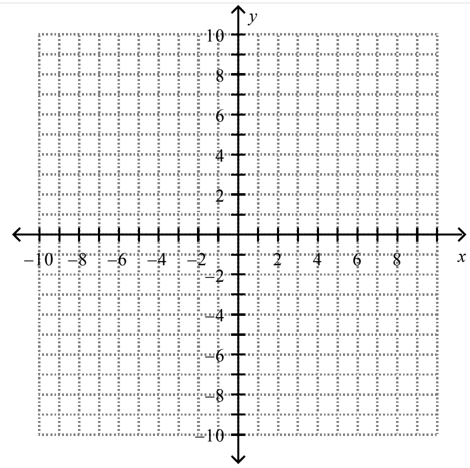
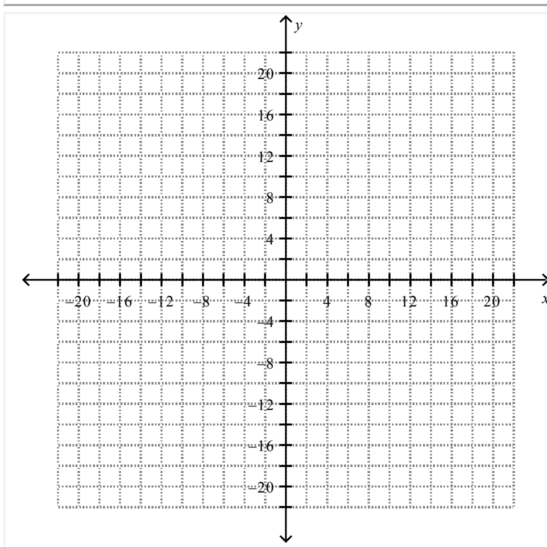
4. reverse the coordinates for the x and y values and plot $y = log_5 x$



When the log function gets more complicated, you will need to use transformations.

| | |
|---|---|
| Parent function | $y = log_b x$ |
| stretch a>1<br>shrink 0<a<1 | $y = alog_b x$<br>**big a** the curve stretches or gets larger, vs. **fraction a** curve gets very thin |
| shift up/down<br>shift left/right | $y = log_b(x - h) + k$<br>where the h is a horizontal shift and the k is our vertical shift |

Graph by first graphing the inverse, and exponential.

| | |
|---|---|
| Graph $log_2 x = y$ |  |
| Graph $log_4(x - 1) = y$ |  |

Graph $log_3(x+1) - 2 = y$