Algebra II

Lesson 7.5: Solving Square Root and Other Radical Equations

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In previous chapters we were solving polynomials: $x^2 - 18 = 0$; $x^2 = 18$; $\sqrt{x^2} = \sqrt{18}$; $x = \pm 3\sqrt{2}$. This is what is called solving a square root equation, because we need to take the square root of x. Now, when we have the variable as a radicand or the variable has a fraction for an exponent (indicates that it is a root), we have what is called a **radical equation**. So to solve for our variable we would have to what???

Solve: $\sqrt{5x+1} - 6 = 0$

$+6 \quad +6$

$(\sqrt{5x+1})^2 = (6)^2$

$5x + 1 = 36$

$-1 \quad -1$

$\frac{5x}{5} = \frac{35}{5}$

$x = 7$

$\square^{1/2(2)}$

Follow the same process we have always used: Start by isolating the variable term. First add/subtract numbers

To get rid of the radical square both sides of the equation and simplify

solve for x

$2 + \sqrt{3x-2} = 6$

$-2 \quad -2$

$(\sqrt{3x-2})^2 = (4)^2$

$3x - 2 = 16$

$+ \quad +2$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

$3\sqrt{5x+1} - 6 = 0$

Coefficient

$+6 \quad +6$

$\cancel{\tfrac{1}{3}} \cdot \cancel{3}\sqrt{5x+1} = 6 \cdot \frac{1}{3}$

$(\sqrt{5x+1})^2 = (2)^2$

Coefficient $5x + 1 = 4$

$5x = 3$

$x = \frac{3}{5}$

We can also solve equation where instead of a radical there is a fractional exponent. How? **Raise each side of the equation to the reciprocal power of the exponent.** With the following rule resulting:

$\frac{3}{4} \cdot \frac{4}{3} = 1 \quad \frac{m}{n} \cdot \frac{n}{m} = 1$

$\frac{2}{3} \cdot \frac{3}{2} = 1$

$$\left(x^{m/n}\right)^{n/m} = |x|$$

- The above rule generates two possible answers: $|x| = \pm$ ⬡
- When both sides of an equation are raised to a power, a chance for extraneous solutions is introduced so…

check solutions to verify presence of an extraneous solution.

A brief review of absolute values:

$| \hexagon | = 5$

$\hexagon = \pm 5$

therefore if $|x+3| = 5$

then: $x + 3 = \pm 5$

and $x + 3 = 5$ *and* $x + 3 = -5$ -3 -3

so! $x = ??$ 2 or -8

The "stuff" inside the AV can either be positive or negative, so there will be 2 possible solutions for x.

The +/- solutions are the 2 possible solutions.

Do both values of *x* work?

Let's see how this works, Solve:

$\frac{1}{2}\ 2\boxed{(x+3)^{\frac{3}{2}}} = 54\ \frac{1}{2}$

$\Rightarrow (x+3)^{3/2 \cdot 2/3} = (27)^{2/3}$ $\quad ((27)^2)^{1/3}$? or $(27^{1/3})^2$ $\downarrow$ $(3)^2$

$|x+3| = 9$

$x+3 = \pm 9$

$x+3 = 9$ or $x+3 = -9$

$x = 6$ or $\cancel{x = -12}$ extraneous

=check $(x+3)^{3/2} = 27$

$(6+3)^{3/2} = 9^{3/2}$ ✓

$(-12+3)^{3/2} = 27$

1. Get rid of coefficient 2 by multiplying by reciprocal.
2. The exponent is $\frac{3}{2}$, so we raise each side by the reciprocal of $\frac{2}{3}$ **rule of exponents:** exponent raised to an exponent, multiply the exponents.
3. Now simplify the right side of the equation the rational exponent means we take the cube root of 27^2 or!! the cube root of 27 then squared.
4. Set the absolute value equal to the $\pm$ the expression and solve

CHECK ANSWERS!! THERE MAY BE A FALSE SOLUTION!!

$\left(\frac{1}{2}\right) 2(x-2)^{2/3} = 50\left(\frac{1}{2}\right)$

$(x-2)^{\cancel{2/3}\ 3/2} = 25^{3/2}$ $\quad (25^{1/2})^3$

$|x-2| = 125$

$x-2 = \pm 125$

$x - 2 = 125$ $+2$ $+2$ $\qquad x-2 = -125$

$x = 127$ $\qquad x = -123$

$3(x+3)^{3/4} - 5 = 76$

$+5$ $+5$

$\left(\frac{1}{3}\right)3(x+3)^{3/4} = 81\left(\frac{1}{3}\right)$

$(x+3)^{3/4 \cdot 4/3} = 27^{4/3}$ $\quad (27^{1/3})^4$

$|x+3| = 81$

$x+3 = -81$ -3 -3 or $x+3 = +81$ -3 -3

$x = \cancel{-84}$ $\qquad x = 78$

$\cancel{(-84+3)^{3/4}}$

no real sol

Equations can contain more than one radical expression

ISOLATE one of the radicals. **IF** it contains an expression with a variable under a radical and a variable outside the radical, focus on isolating the variable under the radical by squaring each side. Well! Well! You may end up with a quadratic equation! You can handle that! ***ALWAYS CHECK FOR EXTRANEOUS SOLUTIONS, PLEASE!***

Now Try:

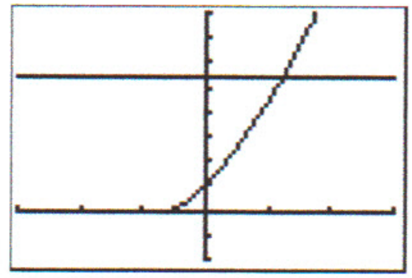
$\sqrt{3x+2}-\sqrt{2x+7}=0$ $+\sqrt{2x+7}\quad +\sqrt{2x+7}$ $(\sqrt{3x+2})^2=(\sqrt{2x+7})^2$ $3x+2=2x+7$ $x=5$ $\sqrt{17}-\sqrt{17}=0$ ✓	$(2x+1)^{1/3}-(2+3x)^{1/3}=0$ $[(2x+1)^{1/3}]^3=[(2+3x)^{1/3}]^3$ $2x+1=2+3x$ ↓ Solve for x $-1=x$
$\sqrt{x+7}-x=1$ $+x\quad +x$ $(\sqrt{x+7})^2=(x+1)^2$ $(x+1)(x+1)$ $x+7=x^2+2x+1$ $-x\ -7\quad -x\ -7$ $x^2+x-6=0$ $(x-2)(x+3)=0$ $\quad (-2)(3)$, $-2+3$ $x-2=0$ $\quad x+3=0$ $x=2$ $\quad \cancel{x=-3}$ $\sqrt{2+7}-2=1$ $\quad \sqrt{-3+7}-(-3)=1$ $\sqrt{9}-2=1$ $\quad \sqrt{4}+3=1$ $3-2=1$ $\quad 2+3=1$ $1=1$	$(2x+1)^{0.5}-(3x+4)^{0.25}=0$ ADD $(2x+1)^{0.5}=(3x+4)^{0.25}$ $((2x+1)^{1/2})^4=(3x+4)^{\cancel{1/4}\,\cancel{4}}$ $(2x+1)^2=3x+4$ $(2x+1)(2x+1)=3x+4$ $4x^2+4x+1-3x-4=0$ $4x^2+x-3=0$ $(4x-3)(x+1)=0$ $x=3/4$ $\quad \cancel{x=-1}$ extraneous solution

If a calculator is available, checking your work will be quicker, but you need to do this without the assistance of technology. Enter each side of the original equation as a separate entry in your calculator's **Y=** function. Locate the intersection of the two lines to verify the solutions.

For the above example we find that there is only one solution for Y=27 as shown on the following graph:

Graph the equation $(x+3)^{\frac{3}{2}} = 27$
on calculator enter

Y1=$(x+3)^{\frac{3}{2}}$
Y2= 27
WINDOW Xmin= -15
Xmax= 15
Scl= 5
Ymin= -10
Ymax= 40
Scl=5



set windows to include y=27 on the graph
we know that $x = 6\ or - 12$ so set windows accordingly.