Roots or radicals are the opposite operation of exponents. You can "undo" a power with a radical, and a radical can "undo" a power. For instance, if you square 2, you get 4, and if you "take the square root of 4", you get 2; if you square 3, you get 9, and if you "take the square root of 9", you get 3.

\[ 4^2 = 16 \quad \text{and} \quad \sqrt{16} = \sqrt{4^2} = 4; \quad 8^2 = 64 \quad \text{and} \quad \sqrt{64} = \sqrt{8^2} = 8; \quad \sqrt{81} = \sqrt{9^2} = 9; \]

\[ \sqrt{144} = \text{________________________} \]

**Question:** What if it is not a perfect square? Look for perfect squares, simplifying the perfect square and leave the rest behind under the radical:

\[ \sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{4^2 \cdot 2} = 4\sqrt{2} \]

\[ \sqrt{180} = \text{________________________} \]

**Question:** What if the radical has a number in the corner? Radicals other than a square root: we also see radicals that have a number in the corner of the radical sign. This is called the **index**: \( \sqrt[n]{\text{ }} \) the index tells us how many times a number must be multiplied by itself under the radical, so to simplify.

\( \sqrt[3]{8} \) we look for a number multiplied by itself 3 times so: \( \sqrt[3]{8} = \sqrt[3]{2^3} = 2 \)

We also must think about variables under the radical too. \( \sqrt[3]{x^3} = x; \quad \sqrt[3]{8x^6} = \sqrt[3]{2^3(x^2)^3} = 2x^2; \)

\( \sqrt[4]{64x^4} = \text{______________}; \quad \text{careful not a perfect cube: } \sqrt[4]{54y^7} = \text{______________} \)

\( \sqrt[4]{16} \) we look for a number multiplied by itself 4 times, so: \( \sqrt[4]{16} = \sqrt[4]{2^4} = 2, \) and don’t forget about those variables! \( \sqrt[4]{81y^{12}} = \sqrt[4]{3^4(y^3)^4} = 3y^3; \)

remembe\d{r} that we may not have a number that can be multiplied by itself 4 times so there will be extra nubers left behind under the radical: \( \sqrt[4]{162t^{15}} = \sqrt[4]{3^42(t^3)^4t^3} = 3t^3\sqrt[4]{2t^3}; \)

\( \sqrt[4]{144x^5y^{12}} = \text{________________________} \)

**Question:** What do we do with radicals when they are combined with a fraction? First look at using the quotient property that allows us to combine the fraction under one radical and if possible simplify canceling out the denominator with factors from the numerator:

\[ \frac{\sqrt[4]{20x^{17}}}{\sqrt[4]{5x^7}} = \sqrt[4]{\frac{20x^{17}}{5x^7}} = \sqrt[4]{4x^{10}} = \sqrt[4]{(2x^5)^2} = 2x^5; \]

\[ \frac{\sqrt[5]{72x^{12}y^{14}}}{\sqrt[5]{2x^4y^2}} = \text{________________________} \]

**Question:** What is rationalizing a denominator? When we get a fraction that has a radical in the denominator we need to multiply the denominator by another radical so that the radical simplifies to a rational denominator:

\[ \frac{\sqrt{3x}}{\sqrt{2}} \quad \text{and} \quad \frac{\sqrt{6x}}{\sqrt{2}} = \frac{\sqrt{6x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6x} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{12x}}{2} \]

if you have a cube root in the denominator, deal with in a similar way:

\[ \frac{\sqrt[3]{5x}}{\sqrt[3]{2}} \quad \text{and} \quad \frac{\sqrt[3]{5x^2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{5x^2}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{10x}}{2} \]