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Algebra II
Lesson 7-7

## Inverse Relations and Functions

A function, as we know, takes a situation described by an equation, plugs in a number for $x$, the independent variable and creates an output. Well, an inverse function takes this output answer performs some operations on it such that we arrive back at our original starting value of $x$. A function can be described as a "DO" and the inverse can be described as the "UNDO." Put another way, an inverse relation is an exact opposite of what a function does and has a special symbol; $\boldsymbol{f}^{-\mathbf{1}}(\boldsymbol{x})$. There are 2 basic steps to formulating an inverse relation.

- Step $1 \quad$ Switch the $x$ and $y$
- Step 2 Solve for the new " y ", and replace y with $\mathrm{f}^{-1}(x)$

Example: Find the inverse function for

$$
\begin{array}{cl}
y=x+3 & \\
x=y+3 & \text { Step 1: switch } \mathrm{x} \text { and } \mathrm{y} \\
x-3=y & \text { solve } \\
x-3=f^{-1}(x) & \text { Step 2: replace } y \text { with } f^{-1}(x)
\end{array}
$$

Sometimes you will be asked to plot a set of ( $x, y$ ) coordinates and then graph the inverse. Understand, the inverse of a function has all the same points as the original function, except that the $\boldsymbol{x}$ 's and $\boldsymbol{y}$ 's have been switched. For instance, supposing your function is made up of these points: $\{(1,0),(-3,5),(0,4)\}$. Then the inverse is given by this set of point: $\{(0,1),(5,-3),(4,0)\}$. In this case simply switch the order of the $x$ and $y$ values.

Example: Plot the data given. Determine the inverse and plot the inverse.

| $x$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $y$ | 0 | 1 | 4 | 9 |$\longrightarrow$| $x$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |

Notice that a straight line passes through the center of both data sets. This line $y=x$, is called the axis of symmetry. In future lessons we will learn that not all functions will necessarily have an inverses. For now, assume that all given function shave a unique inverse.


Example: Given $f(x)=2(x-3)+1$ Find $f^{-1}(x)$, graph both $f(x)$ and $f^{-1}(x)$
And state the domain and range for both
$f(x)$ and $f^{-1}(x)$

Step 1: switch $x$
and y Solve

Step 2: replace $y$ with $f^{-1}(x)$


Note: Watch out for domain restrictions and be able to state the restrictions.
If $x$ is $y$ and $y$ is $x$, then the domain of $f^{-1}$ is the range of $f$, and the range of $f^{-1}$ is the range of $f$.

Now try: For the previous equation, $f(x)=2(x-3)+1$ find $f\left(f^{-1}(4)\right)$.

What happened? When we evaluated the function of the inverse for a given value of $x$, we got the same number. This is an important property that can be used to determine if two equations are in fact the inverse of each other.

$$
\begin{aligned}
& \text { If } f \text { and } f^{-1} \text { are inverse functions, then: } \\
& \left(f \text { o } f^{-1}\right)(x)=x \text { and }\left(f^{-1} o f\right)(x)=x
\end{aligned}
$$

Example: Given $f(x)=3 x^{2} / 4$ and $g(x)=2 \sqrt{x / 3}$ evaluate $\left(f o g^{-1}\right)(12)=$ $\qquad$
What does our answer tell us about $f(x)$ and $g(x)$ ?

