## Lesson 9.4

## Operations with Functions

Functions may be added, subtracted, multiplied, and divided. That is: $f(x)+g(x)$ is a legitimate action. An airplane for example has an airspeed of $415 \mathrm{mph} ; f(x)=415 x$ represents the distance traveled in $x$ hours. With winds of let's say 30 mph we get $g(x)=30 x$. By combining these equations: $f(x)+g(x)=415 x+30 x$, we get a model of the distance the plane will travel in 30 mph winds.

## Function Operations:

$$
\begin{array}{ll}
(f+g)(x)=f(x)+g(x) & (f-g)(x)=f(x)-g(x) \\
(f \cdot g)(x)=f(x) \cdot g(x) & (f \div g)(x)=f(x) \div g(x)
\end{array}
$$

Our math operations hold true for functions. The domain of $x$ will be that of $f$ and $g$. Understand that the more restrictive domain will override. Also, keep in mind that for division, a restriction will be that $g(x) \neq 0$.

If we can combine functions, why not put one function inside another? The process of putting one function inside another is called "composition of functions." The symbol, $o$, is used to show composition. In mathematical terms, $f$ o $g=f(g(x))$. This means that the function $g(x)$ was put inside the function, $f(x)$.

For example, Joe earns a salary of $x$ dollars a month. Taxes and other deductions, which total $\$ 550$, are taken out and are modeled by: $h(x)=x-550$. This new total $h(x)$ is after tax dollars, or take home pay. From his take home pay, Joe has to pay bills totaling $\$ 940$, these include utilities, food, rent, etc. with an equation of $g(x)=h(x)-$ 940,.that is, take home pay less expenses In the composition format, $\mathrm{h}(\mathrm{x})$ is put into $\mathrm{g}(\mathrm{x})$ to find how much Joe will have left after taxes and bills are paid. Mathematically, $g$ o $h(x)=g[h(x)]$. Let's consider the case where Joe earns $\$ 1800$ a month. How does the composition function work?

$$
\begin{aligned}
g o h(x) & =g[h(x)] \\
h(1800) & =g[h(1800)] \\
& =g(1800-5 \\
& =g(1250) \\
& =1250-940 \\
& =310
\end{aligned}
$$

$$
g \text { o } h(1800)=g[h(1800)] \quad \text { where } x=1800
$$

$$
=g(1800-550) \quad \text { where }: h(x)=x-550
$$

where: $g(x)=h(x)-940$
$\$ 310$ is left in the bank

## Example:

Given: $f(x)=x^{2}+3 x-2$ and $g(x)=2 x-5$
find $f$ o $g(-3)=f(g(-3))$

1. Work from the inside out.
2. Solve for $g(-3)$ first
3. The solution for $g(-3)$ is plugged into $f(x)$ and solved for $f(x)$ :
