## Algebra II <br> Lesson 7-1: Roots and Radical Expressions <br> Mrs. Snow Instructor

In Chapter 7 we are going to study roots and radical expressions. Remember that roots or radicals are the inverse (opposite) of applying exponents or powers. You can undo a power with a radical and you can undo a radical with a power. For example $2^{2}=4$ and $\sqrt{4}=\sqrt{2^{2}}=2$.
The square root of a number $a$ is a number $y$ such that $y^{2}=a$. Cube roots, $4^{\text {th }}$ roots, etc. may also be treated in a similar fashion. We also have a definition:

$$
\text { if } a^{n}=b \text {, then } a \text { is the nth root of } b \text {. }
$$

Example: $2^{2}=4,2$ is the square root of 4
$2^{3}=8,2$ is the cube root of 8
$2^{4}=16,2$ is the 4 th root of 16
$2^{5}=32,2$ is the 5 th root of 32

Recall that the square root of 25 is written as $\sqrt{25}$; and in fact has two possible roots $\pm 5$, because $25=(+5)^{2}=$ $(-5)^{2}$. The principal root (the one that is positive) is 5 . Now the cube root of 27 is written as $\sqrt[3]{27}=3$ and only $3^{3}$ can be the solution. The fourth root of 16 is written as $\sqrt[4]{16}= \pm 2$ because $16=(+2)^{4}=(-2)^{4}$. Again the principal root is the positive value, 2 .

## RULES:

Even roots $(\sqrt[2]{x}, \sqrt[4]{x}, \sqrt[6]{x}, \ldots) \quad 2$ solutions (roots): form of $\pm$
Calculator: the answer shown will only be the principal root. REMEMBER: the accurate solution will be the conjugates,$\pm$.
Odd roots $(\sqrt[3]{x}, \sqrt[5]{x}, \sqrt[7]{x}, \ldots) \quad 1$ solution (root): either + or -
Why? A positive or negative number multiplied by itself an even number of times yields a positive number. A negative number multiplied by itself an odd number of times yields a negative number.

## CAUTION!

$>$ If we are only simplifying a radical, the correct answer is the principal root.
$>$ When we set about to solve an equation we need to remember to put a $\pm$ in front of our answer!
$>$ The textbook refers to real number roots. Here we assume the positive root.
$>$ Two real-number roots would be considered the positive and negative roots.

Some roots should be straight forward to solve by virtue of your knowledge of the multiplication tables. Rewrite the number or variable in terms of the nth root using Algebra I exponent rules.
When you get to a root that you may be uncertain of, use your calculator to check if the radicand (stuff under radical) is a perfect root:

Example: find each real -number root, with no calculator:

| $\sqrt{25}$ | $\sqrt{-16}$ | $\sqrt[3]{-27}$ |
| :--- | :--- | :--- |



When we introduce a variable under the radical we get a rule we must remember to follow:

The $\underline{n t h}$ root rule: For any negative real number $a, \sqrt[n]{a^{n}}=|a|$, when $n$ is even.

When you have a variable raised to an even power and under a radical that has an even index, your answer must be in the absolute value form.

Why??

$$
\sqrt{x^{6}}=x^{3}
$$

Evaluate the above equation : for $x=-2$

Yes, we can agree this equation is true, but.... substitute -2 in for $x$ and solve this is a false statement

Now evaluate with the absolute value symbol as the rule states
yep! a true statement as we are looking at the principal root

Even though 64 has two square roots, -8 and 8 , as stated earlier, the $\sqrt{ }$ indicates only the positive root.
REMEMBER: an even index (radical) with an even power variable under the radical will be simplified with answer in absolute value form.

## Simplify:

| $\sqrt{9 x^{6}}$ | rewrite radicand such that you have perfect squares <br> where possible <br> the exponent of $x$ is 6/even so we use our nth root rule <br> to solve this radical expression |
| :--- | :--- |
| $\sqrt[3]{8 b^{6}}$ | rewrite the radicand such that you have the exponent 3 <br> where possible <br> while the exponent of our variable is even the index <br> (cube root) is odd therefore if $b$ is negative then the root <br> must also be negative. |


| $\sqrt{4 x^{2} y^{4}}$ | rewrite the radicand such that you have exponents <br> equal to the index <br> note the even exponents for the variables, remember <br> the nth root rule |
| :--- | :--- |
| $\sqrt[3]{\frac{8}{216}}$ | $\sqrt[4]{x^{8} y^{12}}$ |

