**Algebra II**
**Lesson 6.5/6.6 Theorems about Polynomial Functions**
**Part I**
**Mrs. Snow, Instructor**

The sections of 6.5 and 6.6 are being combined into a single topic with two parts. Part I will cover theorems that will help us to determine all possible roots when only some of the roots are known.

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| **Irrational Root Theorem:** Given a polynomial with rational coefficients and $\sqrt{b}$ is irrational. If of $a+\sqrt{b}$ is a root, then you will also have the root of $a-\sqrt{b}$. ; these are called ***conjugates*** |
| **Imaginary Root Theorem:** If the imaginary number of $a+bi$ is a root of a polynomial with real coefficients, then the conjugate, $a-bi$ is also a root. Again note these are ***conjugates.*** These roots are called <u>complex conjugates</u> |

To find these roots, you will reduce the polynomial down to linear factors and a quadratic factor by dividing the real factors into the original polynomial. When you get a quadratic factor, you will use the Quadratic Formula to solve.

*A polynomial equation has the following roots, find two additional roots.*

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| $2-\sqrt{7}\ and\ 3+2\sqrt{6}$ | $1+\sqrt{3}, and\ -\sqrt{11}$ |
| $3-i\quad and\quad 2i$ | $12+3i,\quad and\ 4-i$ |

*Find a 3rd degree polynomial equation with rational coefficients that has the given roots*

| $1 \ and \ 3i$ | $2 + i \quad and - 3$ |
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| | |

Find a 4th degree polynomial equation with rational coefficients that has the given roots:

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**Fundamental theorem of Algebra**

In 1797 Carl Gauss proved what is known as the **Fundamental Theorem of Algebra.** It states that *a polynomial of degree* $n \geq 1$ *has at least one complex zero. In essence, t*he fundamental theorem of algebra guarantees that every polynomial has a complete factorization, if we are allowed to use complex numbers $(a + bi)$. Remember that a real number may be written as a complex number.

**Fundamental Theorem of Algebra;** *translation!:* An nth degree polynomial equation has exactly n roots; they may be rational, irrational, or complex.

Find the number of complex roots, and the possible number of real roots

$$x^5 + x^3 + 2x^2 - 6 = 0$$

$$x^{10} + x^8 + x^4 + 3x^2 - x + 1 = 0$$