Algebra 2
Lesson 6-3: Dividing Polynomials
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Dividing two numbers we use a process known as long division.

\[
\begin{array}{c}
1512 \div 4 \\
\underline{\phantom{1512}} \\
-1512 \\
-1248 \\
\underline{\phantom{-1512}} \\
-264 \\
-248 \\
\underline{\phantom{-1512}} \\
-16 \\
\end{array}
\]

\[
(378)(4) = 1512
\]

\[
\begin{array}{c}
1649 \div 7 \\
\underline{\phantom{1649}} \\
-1649 \\
-147 \\
\underline{\phantom{-1649}} \\
122 \\
14 \\
\underline{\phantom{-1649}} \\
65 \\
2 \\
\underline{\phantom{-1649}} \\
65 \\
-64 \\
\underline{\phantom{-1649}} \\
1 \\
\end{array}
\]

OR

\[
\begin{array}{c}
235 \\
\underline{\phantom{235}} \\
164 \\
-14 \\
\underline{\phantom{235}} \\
21 \\
14 \\
\underline{\phantom{235}} \\
7 \\
2 \\
\underline{\phantom{235}} \\
5 \\
\underline{\phantom{235}} \\
4 \text{ Remainder}
\end{array}
\]

We can also divide polynomials:

\[
\begin{array}{c}
x^2 + 3x - 18 \div x - 3 \\
\underline{\phantom{x^2 + 3x - 18}} \\
(x - 3)(x + 6) = x^2 + 3x - 18
\end{array}
\]

1. Look at the first term in each polynomial. Here, ask, \( x \) goes into \( x^2 \) how many times?
2. As with long division, multiply quotient by the divisor, and simplify; drop the next term from the dividend.
3. Repeat the process of bringing down the next term followed by dividing, multiplying, and subtracting.

\[
\begin{array}{c}
x^2 + 2x - 30 \div x - 5 \\
\underline{\phantom{x^2 + 2x - 30}} \\
(x - 5)(x + 6) = x^2 + 2x - 30 + 5 \text{ Remainder}
\end{array}
\]

\[
\begin{array}{c}
x^3 + 7x^2 - 4 + x + 2 \\
\underline{\phantom{x^3 + 7x^2 - 4 + x + 2}} \\
(x + 2)(x^2 + 5x - 10) + 16
\end{array}
\]
When there is a remainder, the proper form for the factor is:

\[(\text{dividend}) = (\text{divisor})(\text{quotient}) + \text{remainder}\]

How does this dividing help us?
1. Given a factor, we can simplify by dividing to find the factor pair.
2. We can verify if a polynomial is a factor of another polynomial. If the remainder is zero then our divisor is a factor!

**Remainder Theorem**

If we have a polynomial \(P(x)\) and it is divided by \(x - a\), then:

\[P(a) = \text{number} = \text{remainder}\]

A second type of division we can use which is quicker than long division is known as **synthetic division**. This technique works only when we have a linear binomial in the form of \(x - a\), that is \(x - [a]\)

\[
\begin{array}{c|ccc}
& x^3 & -7x^2 & +15x - 9 + x - 3 \\
\hline
a = 3 & 1 & -7 & 15 & -9 \\
\hline
3 & & 3 & -12 & 9 \\
\hline
& 1 & -4 & 3 & 10 \\
\end{array}
\]

\(P(a) = \text{remainder} = 0\)

So \((\text{linear})(\text{quadratic}) = \text{cubic}\)

\((x-3)(x^2 - 4x + 3) = x^3 - 7x^2 + 15x - 9\)

\[
\begin{array}{c|cccc}
& x^3 & +4x^2 & +x & - 6 + x + 1 \\
\hline
a = -1 & 1 & 4 & 1 & -6 \\
\hline
-1 & & 1 & -3 & 2 \\
\hline
& 1 & -2 & 1 & -1 \\
\end{array}
\]

\(P(a) = \text{remainder} = 0\)

\((x+1)(x^2 + 3x - 2) + 4\)

**Is \(x + 2\) a factor of:**

\[
\begin{array}{c|cc}
& 2x^2 & +7x + 6 \\
\hline
-2 & 2 & 7 & 6 \\
\hline
& 2 & 3 & 10 & \text{no remainder} \\
\end{array}
\]

\(\text{Yes } (x+2) \text{ is a factor}\)

\[
\begin{array}{c|cccc}
& x^3 & -5x & -10 \\
\hline
-2 & 1 & 0 & -5 & -10 \\
\hline
-2 & -4 & 2 & \text{no remainder} \\
\hline
1 & -2 & 1 & -5 & \text{no remainder} \\
\end{array}
\]

\(\text{Not fact}\)
The volume in cubic feet of a workshop's storage chest can be expressed as the product of its three dimensions by the given function: \( V(t) = x^3 + 7x^2 + 10x \). The depth of the chest is given by the function \((x + 2)\). Find the linear expressions for the other two dimensions.

\[
\begin{align*}
0 &= -2x^3 + 7x^2 + 10x \\
-2 \left[ 
\begin{array}{cccc}
1 & 7 & 10 & 0 \\
\end{array}
\right] &= x + 2 \\
\hline 1 & 5 & 0 & 0 \\
\end{align*}
\]

\( V(x) = (x+2)(x^2+5x+0) = (x+2)(x^2+5x) = (x+2)(x)(x+5) \)

Other dimensions

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Find \( P(4) \) for \( P(x) = x^4 - 5x^2 + 4x + 12 \) use synthetic division:

\[
\begin{array}{rcccc}
-4 & 1 & 0 & -5 & 4 & 12 \\
\hline
1 & 4 & 11 & 48 & 204 \\
\end{array}
\]

\( P(4) = 204 \)

Now solve for \( P(-4) \):

\[
P(4) = 4^4 - 5(4^2) + 4(4) + 12 \\
= 256 - 80 + 16 + 12 \\
= 204
\]

Same!

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Find \( P(-1) \) for \( P(x) = 2x^4 + 6x^2 - 5x^2 + 60 \)

\[
\begin{array}{rcccrcccc}
-1 & 2 & 6 & -5 & 0 & 60 \\
\hline
2 & 4 & -9 & 9 & 51 \\
\end{array}
\]

\( P(-1) = 5 \)

\[
P(-1) = 2(-1^4) + 6(-1^3) - 5(-1^2) + 60 \\
= 2 - 6 - 5 + 60 \\
= 51
\]

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