## Algebra II

## Lesson Quadratic Inequalities

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Suppose that $y=a x^{2}+b x+c$. If the equal sign is replaced with an inequality we have what is called a quadratic inequality.

One-Variable Inequality:

- $a x^{2}+b x+c>0(y>0)$ The solution includes all $x$-values, where the graph of $y$ is above the $x$-axis.
- $a x^{2}+b x+c<0(y<0)$ The solution includes all $x$-values, where the graph of $y$ is below the $x$-axis.




## Example

Solve: $x^{2}-x-12>0$

$$
\begin{gathered}
x^{2}-x-12=0 \\
(x+3)(x-4)=0 \\
x=-3 \quad x=4
\end{gathered}
$$



|  | -4 | 0 | 5 |
| :--- | :---: | :---: | :---: |
| $x+3$ | - | + | + |
| $x-4$ | - | - | + |
| product | + | - | + |

want $>0$ or product of factors will be + Answer:

$$
x<-3 \text { or } x>4
$$

Interval notation: $(-\infty,-0) \cup(4, \infty)$
graphing the quadratic we see:

Where is the parabola above the $x$-axis?

1. Replace inequality with an equal sign.
2. Factor the quadratic.
3. The solutions to the equation are the boundaries for the intervals that are the solutions to the inequality.
4. Make a number line, using the boundary numbers to separate sections of the number line.
5. Make a table of the factors vs. a test point on a number line to determine which segment(s) work for a solution for the inequality.
6. Solution segment(s) are where the test point yields correct sign for the product of the factors.

We want $>0$ so the solutions are as show highlighted. With the > sign recognize that the end points are not part of the solution.


While the graphing is easy and thus visually we can see the solution areas. To better prepare us for precalculus, we need to understand how to solve inequalities algebraically.

| Solve: $\quad x^{2}-4 x-5<0$ | Solve: $\quad x^{2}-2 x-8 \leq 0$ |
| :--- | :--- | :--- |

## Applications

An object is launched at 4.9 meters per second from a 58.8 -meter tall platform. The equation for the object's height at time $t$ seconds after launch is $s(t)=-4.9 t^{2}+4.9 t+58.8$, where $s$ is in meters. When does the object hit the ground?

An object is launched directly upward at 64 feet per second from a platform 80 feet high. The equation for the object's height is $h(t)=-16 t^{2}+64 t+80$.
a) At how many seconds will the object have a height of 100 feet?
b) There are 2 answers. Why?

An object is launched from ground level directly upward at a rate of 144 feet per second. The equation for the object's height is $y=-16 x^{2}+144 x$.
a) What values of $x$ is the object at OR ABOVE a height of 288 feet?
b) How long is the object at or above this height?

The area of a rectangle is 20 square inches. The length is 4 more than three times the width. Find the length and width of the rectangle. (Hint: draw a picture \& set up a system of equations.)

