Suppose that \( y = ax^2 + bx + c \). If the equal sign is replaced with an inequality we have what is called a quadratic inequality.

One-Variable Inequality:
- \( ax^2 + bx + c > 0 \) (\( y > 0 \)) The solution includes all \( x \)-values, where the graph of \( y \) is above the \( x \)-axis.
- \( ax^2 + bx + c < 0 \) (\( y < 0 \)) The solution includes all \( x \)-values, where the graph of \( y \) is below the \( x \)-axis.

Example

Solve: \( x^2 - x - 12 > 0 \)

\[
\begin{align*}
x^2 - x - 12 &= 0 \\
(x + 3)(x - 4) &= 0 \\
x &= -3 \quad x = 4
\end{align*}
\]

\[
\begin{array}{ccc}
-4 & 0 & 5 \\
x + 3 & - & + \\
x - 4 & - & - \\
\text{product} & + & + \\
\end{array}
\]

Want \( > 0 \) or product of factors will be +

Answer: \( x < -3 \) or \( x > 4 \)

Interval notation: \((-\infty, -3) \cup (4, \infty)\)

graphing the quadratic we see:

Where is the parabola above the \( x \)-axis?
While the graphing is easy and thus visually we can see the solution areas. To better prepare us for precalculus, we need to understand how to solve inequalities algebraically.

\[
\text{Solve: } \quad x^2 - 4x - 5 \leq 0
\]
\[
(x-5)(x+1) = 0
\]
\[
x = 5 \quad x = -1 \quad \therefore \quad \text{Boundaries}
\]

Test points:

\[
\begin{array}{c|c|c|c}
\hline
x & -2 & 0 & 5 \\
\hline
(x-5)(x+1) & - & - & + \\
\hline
\end{array}
\]

Solutions \(x < 0\) are negative so...

Solution: \(1 \leq x \leq 5\)

\[
\text{graph: } \quad \downarrow \quad \text{interval notation: } \quad (-1, 5)
\]

\[
\text{Solve: } \quad x^2 - 2x - 8 \leq 0
\]
\[
(x-4)(x+2) = 0
\]
\[
x = 4 \quad x = -2 \quad \therefore \quad \text{Boundaries}
\]

Test points:

\[
\begin{array}{c|c|c|c}
\hline
x & -3 & 0 & 5 \\
\hline
(x-4)(x+2) & - & - & + \\
\hline
\end{array}
\]

Solutions are \(x \leq 4\)

\[
\text{graph: } \quad \downarrow \quad \text{or } [-2, 4]
\]

\[
\text{Solve: } \quad -2x^2 - 6x + 20 \leq 0
\]

Factor out -2

\[
-x^2 - 3x - 10 \geq 0
\]

\[
(x+5)(x-2) \geq 0
\]

\[
x = -5 \quad x = 2 \quad \therefore \quad \text{Boundaries}
\]

Test points:

\[
\begin{array}{c|c|c|c}
\hline
x & -6 & 0 & 3 \\
\hline
(x+5)(x-2) & - & - & + \\
\hline
\end{array}
\]

Solution: \((-\infty, -5] \cup [2, \infty)\)

Graphing, we can see both original equation and factored equation yield same answer.

\[
\text{graph: } \quad \downarrow \quad \text{or } [-2, 4]
\]

\[
\text{graph: } \quad \downarrow \quad \text{or } [-2, 4]
\]
Applications

An object is launched at 4.9 meters per second from a 58.8-meter tall platform. The equation for the object's height at time t seconds after launch is \( s(t) = -4.9t^2 + 4.9t + 58.8 \), where \( s \) is in meters. When does the object hit the ground?

\[ \text{time for a height } s = 0 \]
\[ 0 = -4.9t^2 + 4.9t + 58.8 \]
\[ 0 = -4.9(t^2 - t - 12) \]
\[ 0 = (t - 4)(t + 3) \]
\[ t = 4 \quad t = -3 \] \( \text{no negative time} \)

Ans: \( t = 4 \) sec.

With calculator:

1. \text{2nd Trace 2: \textit{enter}}
2. \( (2 \rightarrow 10^5) \)
3. \( x = 4 \implies 4 \text{ seconds} \)

An object is launched directly upward at 64 feet per second from a platform 80 feet high. The equation for the object's height is \( h(t) = -16t^2 + 64t + 80 \).

a) At how many seconds will the object have a height of 100 feet?

\[ -16t^2 + 64t + 80 = 100 \]
\[ -16t^2 + 64t - 20 = 0 \]

\[ \text{OR} \]

\[ \text{graph } y_1 = -16t^2 + 64t + 80 \]
\[ y_2 = 100 \]

and find intersections

b) There are 2 answers. Why?

\[ t = 1.34 \text{ sec} \]

or

\[ t = 3.66 \text{ sec} \]

Object is launched, goes up into the air, passes 100 ft and comes down, passing 100 ft a second time.
An object is launched from ground level directly upward at a rate of 144 feet per second. The equation for the object's height is \( y = -16x^2 + 144x \).

a) What values of \( x \) is the object at OR ABOVE a height of 288 feet?

\[
y_1 = -16x^2 + 144x \\
y_2 = 288
\]

Window: \( y_{\text{min}} = -10 \), \( y_{\text{min}} = -10 \)
\( y_{\text{max}} = 350 \), \( y_{\text{max}} = 350 \)
\( v_{\text{sel}} = 1 \), \( y_{\text{sel}} = 50 \)

Object at or above 288 ft:
\( 3 \text{ sec} \leq x \leq 5.74 \text{ sec} \)

b) How long is the object at or above this height?

\[5.74 - 3 = 2.74 \text{ sec}\]

The area of a rectangle is 20 square inches. The length is 4 more than three times the width. Find the length and width of the rectangle. (Hint: draw a picture & set up a system of equations.)

\[ (l)(w) = \text{area} \]
\[ l = 4 + 3w \]

\[ (4 + 3w)(w) = 20 \]
\[ 4w + 3w^2 - 20 = 0 \]
\[ 3w^2 + 4w - 20 = 0 \]
\[ 3w^2 - 6w + 10w - 20 = 0 \]
\[ 3w(w - 2) + 10(w - 2) \]
\[ (3w + 10)(w - 2) = 0 \]
\[ 3w + 10 = 0 \quad w - 2 = 0 \]
\[ w = -\frac{10}{3} \quad \text{false} \]
\[ w = 2 \]

\[ l = 4 + 3(2) = 10 \text{ in} \]