## Algebra 2

## Lesson 5-4: Factoring Quadratic Expressions <br> Mrs. Snow, Instructor

Factoring is a way we break down a number or expression into a product of its factors or factor pairs.

Example: write the factor pairs for 12: $1 \cdot 12,2 \cdot 6$, and $\qquad$
$\qquad$
In a quadratic equation we can often simplify it by factoring out the greatest common factor, GCF.

| $5 x^{2}+20 x-25$ | $7 p^{2}+21 p$ | $s^{3}+2 s^{2}-7 s$ |
| :---: | :---: | :---: |

Factoring a quadratic is also possible. Remember, in Alg I you took 2 binomials and multiplied or FOIL to get the quadratic, well here we take the quadratic and break it down into the binomial pairs. Given a trinomial quadratic in the form of $a x^{2}+b x+c$, it may be possible to break it down into two binomial expressions
$x^{2}+b x+c \quad$ when " $c$ " is positive:
find two numbers that will multiply to equal the constant term " c ", and add up to equal " b "

| $x^{2}+10 x+24$ |  |
| :--- | :--- |
| $(x)$ | $)(x$ |$\quad$| 1. Make a list of all the factors of 24 |
| :--- |
| 2. Which factor pairs add up to equal the coefficient |
| term? |

3. Now make a "template" of 2 sets of parentheses
4. Recognize that the first term of each binomial will be an $\mathbf{x}$
5. Now you can fill in the constant terms with the 2 values that multiply out to 24 and add up to 10 !
6. CHECK YOUR WORK!!!!!!!

$$
\begin{aligned}
& x^{2}+6 x \\
& \quad+4 x+24= \\
& x^{2}+10 x+24
\end{aligned}
$$

$x^{2}+b x-c \quad$ when " $c$ " is negative: find two numbers that will multiply to equal " $-c$ " but when subtracted will equal "b"


To have a positive 7 we will make 2 negative:

Check:

$$
x^{2}-2 x
$$

$$
+9 x-18=
$$

$$
x^{2}+7 x-18
$$

1. Make a list of all the factors of 18. Recognize that we will be looking at a positive factor and a negative factor! $(-) \times(+)=(-)$.
2. Which factor pair has the difference of 7 ? Then place signs such that the difference is positive 7!
3. Now make a "template" of 2 sets of parentheses.
4. Recognize that the first term of each binomial will be an $\mathbf{x}$.
5. Now you can fill in the constant terms with + and -signs inserting the factor pairs such that the 2 values that multiply out to -18 and have a difference of +7 .

| Factor: | $x^{2}-6 x+8$ |
| :--- | :--- |
|  | $x^{2}-8 x-20$ |
|  |  |

When $a x^{2}+b x+c,($ where $a \neq 1)$

| Factor: $\begin{aligned} & \boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c} \\ & 4 x^{2}+16 x+15 \\ & \downarrow \searrow \end{aligned}$ | 1. Multiply $a$ and $c$ <br> 2. Make a table of the factor pairs of $a c$ <br> 3. Which factor pair when added is equal to the coefficient $b$ ? <br> 4. Rewrite the linear term using the factor pairs of $a$ and $c$. <br> 5. Now cut the quadratic in half <br> 6. Factor the left side and factor the right side separately. <br> 7. WHEN FACTORED, YOU MUST HAVE <br> THE SAME BINOMIALS!! <br> 8. Factor both: factor out the common binomial. <br> 9. You have a factored quadratic! <br> 10. AND check your work! Remember: factor the left, factor the right, and factor both sides! | $\begin{gathered} \begin{array}{c} a=4, c \\ \\ a c \\ \text { Factor pairs: } \end{array} \\ \hline a \times c=60 \\ \hline 1 \cdot 60 \\ 2 \cdot 30 \\ 3 \cdot 20 \\ 4 \cdot 15 \\ 5 \cdot 12 \\ 6 \cdot 10 \end{gathered}$ | 15 , and $b=16$ $\begin{aligned} & (15)=60 \\ & \hline a+c=15 \\ & \hline \text { sum }=61 \\ & \text { sum }=32 \\ & \text { sum }=23 \\ & \text { sum }=19 \\ & \text { sum }=17 \\ & \mathbf{6}+\mathbf{1 0}=\mathbf{1 6} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| First factor out the negative leading coefficient!!!$-4 x^{2}-4 x+15$ |  | $x^{2}-16 x-$ |  |

## Perfect Square Trinomials

$$
a^{2}+2 a b+b^{2}=(a+b)^{2} \quad \text { or } \quad a^{2}-2 a b+b^{2}=(a-b)^{2}
$$

When we square a binomial we will get a quadratic that is in the form of a perfect trinomial square. Study this form so that you will recognize it and take advantage of the short cut.

1) Write the first and third terms as perfect squares
2) Write or attempt to write the second term in the form of $2 \cdot a \cdot b$

$$
x^{2}+8 x+16=
$$

$$
4 x^{2}-12 x+9=
$$

$$
\begin{aligned}
& \text { Difference of Two Squares } \\
& a^{2}-b^{2}=(a+b)(a-b)
\end{aligned}
$$

Factoring Flow Chart for: $1 x^{2}+b x+c$

|  | $\begin{gathered} +\boldsymbol{b} \\ (+ \text { big })(- \text { small }) \\ x^{2}+7 x-18 \\ (x-2)(x+9) \end{gathered}$ | $\underbrace{x^{2}-5 x-14}_{\substack{\boldsymbol{b} \\ (- \text { big })(+ \text { small })}} \begin{gathered} (x-7)(x+2) \end{gathered}$ |
| :---: | :---: | :---: |

Factoring Flow Chart for: $a x^{2}+b x+c$

|  |  |
| :---: | :---: |
| $\begin{array}{rc} \text { (4) }(5)=20, \quad 4+5=9 & (-5)(-12)=60,(-5)+(-12)=-17 \\ \mathbf{2} \boldsymbol{x}^{\mathbf{2}}+\mathbf{9} \boldsymbol{x}+\mathbf{1 0} & \mathbf{4} \boldsymbol{x}^{\mathbf{2}}-\mathbf{1 7} \boldsymbol{x}+\mathbf{1 5} \end{array}$ | $\begin{array}{cc} (-3)(10)=-30,(-3)+(10)=7 & (1)(-8)=-8,(1)+(-8)=-7 \\ \mathbf{5} \boldsymbol{x}^{2}+\mathbf{7} \boldsymbol{x}-\mathbf{6} & \mathbf{2 \boldsymbol { x } ^ { \mathbf { 2 } } - \mathbf { 7 } \boldsymbol { x } - \mathbf { 4 }} \end{array}$ |
| $\begin{array}{cc} 2 x^{2}+4 x+5 x+10 & 4 x^{2}-5 x-12 x+15 \\ 2 x(x+2)+5(x+2) & x(4 x-5)-3(4 x-5) \\ (x+2)(2 x+5) & (4 x-5)(x-3) \end{array}$ | $\begin{array}{lc} 5 x^{2}-3 x+10 x-6 & 2 x^{2}+1 x-8 x-4 \\ x(5 x-3)+2(5 x-3) & x(2 x+1)-4(2 x+1) \\ (5 x-3)(x+2) & (2 x+1)(x-4) \end{array}$ |

